THE STATUS $\alpha_s$ FROM THE LATTICE AND HADRONIC $\tau$ DECAYS

Kim Maltman, York University

CSSM, Adelaide, March 31, 2010

OUTLINE

• Context (including lattice vs. $\tau$ decay tension)

• Updates of the UKQCD/HPQCD lattice approach

• New results on the hadronic $\tau$ decay determination

• Propsectcts/issuess
COLLABORATORS

- CSSM version lattice $\alpha_s$: Derek Leinweber, Peter Moran and Andre Sternbeck

- $\tau$ decay $\alpha_s$: Tzahi Yavin

- work in progress on impact of possible duality violation in $\tau \alpha_s$ (etc.): Maarten Golterman, Santi Peris, Oscar Cata, Matthias Jamin (⋯)
CONTEXT ETC. (AS OF MID-2008)

- HPQCD/UKQCD [PRL95 (2005) 052002]: perturbative analysis of UV-sensitive lattice observables [dominant input to PDG08 assessment $\alpha_s(M_Z) = 0.1176(20)$]

$$[\alpha_s(M_Z)]_{\text{latt}} = 0.1170(12)$$

- Conventional ALEPH, OPAL [e.g., EPJC56 (2008) 305]: “(k,m) spectral weight” hadronic $\tau$ determination:

$$[\alpha_s(M_Z)]_{\tau} = 0.1212(11)$$

- c.f. other recent determinations [TABLE]
<table>
<thead>
<tr>
<th>Source</th>
<th>$\alpha_s(M_Z)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Global EW fit</td>
<td>0.1193(28)</td>
</tr>
<tr>
<td>H1+ZEUS NLO inclusive jets</td>
<td>0.1198(32)</td>
</tr>
<tr>
<td>H1 high-$Q^2$ NLO jets</td>
<td>0.1182(45)</td>
</tr>
<tr>
<td>Non-singlet structure functions</td>
<td>0.1142(23)</td>
</tr>
<tr>
<td>NNLO+NLLA LEP event shapes</td>
<td>0.1224(39)</td>
</tr>
<tr>
<td>NNLO+NLLA JADE event shapes</td>
<td>0.1172(51)</td>
</tr>
<tr>
<td>FNAL inclusive jets, $p\bar{p}$</td>
<td>0.1161(48)</td>
</tr>
<tr>
<td>ZEUS NLO inclusive jets, $\gamma p$</td>
<td>0.1223(38)</td>
</tr>
<tr>
<td>$N^3LL$ ALEPH+OPAL thrust distrib's</td>
<td>0.1172(21)</td>
</tr>
<tr>
<td>NNLO 3-jet rates in $e^+e^-$</td>
<td>0.1175(25)</td>
</tr>
<tr>
<td>$\Gamma[\gamma(1s) \rightarrow \gamma X]/\Gamma[\gamma(1s) \rightarrow X]$</td>
<td>0.1190(60)</td>
</tr>
<tr>
<td>Lattice PS $\bar{c}c$ correlator moments</td>
<td>0.1174(12)</td>
</tr>
<tr>
<td>Lattice V, A 2-point functions</td>
<td>0.1181(13)</td>
</tr>
</tbody>
</table>

**NOTE:** expt'l det’n errors large c.f. nominal lattice, $\tau$

Excluding $\tau$, lattice, Bethke [0908.1135] average

$$\alpha_s(M_Z) = 0.1184(7) \rightarrow 0.1179(13)$$
UPDATES OF HPQCD LATTICE APPROACH

• Based on perturbative analyses of observables, $O_k$, measured on MILC (asqtad) $n_f = 2 + 1$ ensembles

• $O(\alpha_s^3) \ D = 0 \ (m_q = 0)$ expansion

\[
[O_k]_{D=0} = D_k \alpha_T(Q_k) \left[ 1 + c_1^{(k)} \alpha_T(Q_k) + c_2^{(k)} \alpha_T^2(Q_k) + \cdots \right]
\]

with $Q_k = d_k/a$ the BLM scale for $O_k$

• $D_k$, $c_1^{(k)}$, $c_2^{(k)}$, $d_k$: Q. Mason et al. 3-loop lattice PT
• Original HPQCD/UKQCD analysis [PRL 95 (2005) 052002]: $a \sim 0.18, 0.12, 0.09$ fm ensembles

• HPQCD [PRD78 (2008) 114507], CSSM [PRD78 (2008) 114504] updates add new $a \sim 0.15, 0.06$ fm ensembles, one $(am_{\ell}, am_s) a \sim 0.045$ fm ensemble (HPQCD only) (results dominated by finer ensembles)

• $m_q$-dependent NP contributions: linear $m_q$ extrapolation/subtraction

• $m_q$-independent NP: estimate/subtract via LO $\langle aG^2 \rangle$ (+ fitted $D > 4$ for more long-distance-sensitive observables in 2008 HPQCD)
Some relevant details (summary)

- $D = 0$ to $O(\alpha_s^3)$ insufficient to account for observed scale dependence $\Rightarrow$ MUST fit additional HO term(s)

- 2008 HPQCD, CSSM: different $D = 0$ expansion parameter choices $\Rightarrow$ different (complementary) handling of residual HO perturbative uncertainties

- $m_q \to 0$ extrapolation very reliable:
  - many $(am_\ell, am_s)$ for $a \sim 0.12$ fm, very good linearity (plus good linearity for other $a$ as well)
  - extrapolation very stable to added non-linear terms
• Re $m_q$-independent NP subtraction:

  – $\langle aG^2 \rangle = 0 \pm 0.012 \text{ GeV}^4$ (HPQCD), with independent fit for each $O_k$

  – $\langle aG^2 \rangle = 0.009 \pm 0.007 \text{ GeV}^4$ (CSSM), common input for all $O_k$ to identify small NP cases

  – estimated $D = 4$ correction tiny for shortest-distance-sensitive observables (e.g., $\log(W_{11})$, $\log(W_{12})$)

  – After fitted $m_q$-independent NP subtractions, HPQCD observables with LARGE estimated $D = 4$ corrections yield $\alpha_s$ in good agreement with $\log(W_{11})$ etc.
AND IN A BIT MORE DETAIL...

- Original 2005 HPQCD/UKQCD, 2008 HPQCD:
  
  * $r_1, \frac{r_1}{a}, \langle aG^2 \rangle$: independent fit w/ priors for each $O_k$
  
  * $r_1, \frac{r_1}{a}$: small (measured) prior widths $\Rightarrow$ possible unphysical observable-dependence effects small
  
  * Relation of expansion parameter, $\alpha_V$, to $\bar{\alpha}_S^{\overline{MS}}$ unknown beyond 4th order
  
  * $O_k$ with potentially sizeable $m_q$-independent NP subtractions included in analysis
  
  * (2008 update): better agreement of $\langle aG^2 \rangle$ from different $O_k$ when fitted $D > 4$ forms included [HPQCD private communications]
2008 CSSM re-analysis:

* measured $r_1, \frac{r_1}{a}$, charmonium sum-rule $\langle aG^2 \rangle$ (with errors): common, external input for all $O_k$

* LO $D = 4 \langle aG^2 \rangle$ estimate of $m_q$-independent NP contribution/subtraction

* Relation of expansion parameter to $\alpha_s^{MS}$ exactly specified

* focus on $O_k$ where estimated $D = 4$ NP $\langle aG^2 \rangle$ subtraction small, hence $D > 4$ presumably even smaller

More on the two $D = 0$ expansion parameters choices

$D = 0$ expansion parameter $\alpha_T$, $\beta$ function $\beta^T$ to 4-loops from $\beta^{MS} \Rightarrow \beta^{T}_{4,5,...}$ incompletely known
- Expand $\alpha_T$ in $\alpha_0 = \alpha_T(Q_k^{\max})$, $t_k = \log[(Q_k/Q_k^{\max})^2]$

\[
\frac{O_k}{D_k} = \cdots + \alpha_0^4 (c_3^{(k)} + \cdots) + \alpha_0^5 (c_4^{(k)} - 2.87c_3^{(k)} t_k + \cdots) \\
+ \alpha_0^6 \left( c_5^{(k)} - 0.0033\beta^T_4 t_k - 3.58c_4^{(k)} t_k \right) \\
+ [5.13 t_k^2 - 1.62 t_k] c_3^{(k)} + \cdots + \alpha_0^7 \left( c_6^{(k)} - 0.0010 \beta^T_5 t_k + [0.0094 t_k^2 - 0.0065 c_1^{(k)} t_k] \beta^T_4 \\
- 4.30 c_5^{(k)} t_k + [7.69 t_k^2 - 2.03 t_k] c_4^{(k)} \\
+ [-7.35 t_k^3 + 6.39 t_k^2 - 4.38 t_k] c_3^{(k)} + \cdots \right) + \cdots
\]

- Incompletely known $\beta^T_{4,5,\ldots}$ distorts fit parameters
HPQCD approach

* $\alpha_T \rightarrow \alpha_V$ defined such that $\beta^V_4 = \beta^V_5 = \cdots \equiv 0$

* $\Rightarrow$ no distortion of fit parameters

* expansion for $\alpha_V$ in terms of $\alpha_{\overline{MS}}$ in principle well-defined

* (however) expansion coefficients beyond 4th order depend on $\beta_{\overline{MS}}^{4,5,\ldots}$, hence not known

* impact of HO (after fitting $c^{(k)}_{3,4,\ldots}$) localized to conversion/running to $\alpha_{\overline{s}}(M_Z)$
CSSM approach

* $\alpha_T$ defined as 3-order-truncated expansion of $\alpha_V^p$

* $\Rightarrow$ conversion to $\alpha_{\overline{MS}}$ exact but $\beta_{4,5}^T$ depend on $\beta_{4,5}^{\overline{MS}}$, hence incompletely known

* Fit parameter distortions reducible by hand:
  * focus on highest intrinsic scale $O_k$
  * restrict $t_k$ (subset of finest lattices)
  * stability c.f. expanding subset as test
### FULL HPQCD RESULTS

<table>
<thead>
<tr>
<th>Value</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1185(8)</td>
<td>( \log W_{11} )</td>
</tr>
<tr>
<td>0.1185(8)</td>
<td>( \log W_{12} )</td>
</tr>
<tr>
<td>0.1185(8)</td>
<td>( \log W_{BR} )</td>
</tr>
<tr>
<td>0.1184(8)</td>
<td>( \log W_{CC} )</td>
</tr>
<tr>
<td>0.1184(8)</td>
<td>( \log W_{13} )</td>
</tr>
<tr>
<td>0.1184(9)</td>
<td>( \log W_{14} )</td>
</tr>
<tr>
<td>0.1183(9)</td>
<td>( \log W_{22} )</td>
</tr>
<tr>
<td>0.1181(9)</td>
<td>( \log W_{23} )</td>
</tr>
</tbody>
</table>

**Ave (all):** \(0.1183(8)\)

**Ave small NP:** \(0.1185(8)\)

**Large NP subtractions for 3 outliers:**

- Ave (all): \(0.1183(8)\)
- Ave small NP: \(0.1185(8)\)
**COMPARISON OF HPQCD, CSSM RESULTS**

- Results for a selection of three least-NP and four most-NP observables.

- \( \delta_{D=4} \equiv \) fractional change from scale dependence of “raw” observable to that of \( m_q \)-independent NP-subtracted version between \( a \sim 0.12 \) and \( \sim 0.06 \) fm (\( \langle aG^2 \rangle = 0.009 \ GeV^4 \) as input).

- **NOTE:** re estimated NP \( D = 4 \) corrections
  
  * corrections far and away the largest for the 3 HPQCD “outliers”

  * despite *large* corrections, \( \alpha_s \) agree with results from observables where NP corrections negligible.
\(-\delta_{D=4}\) and resulting \(\alpha_s(M_Z)\) values

<table>
<thead>
<tr>
<th>(O_k)</th>
<th>(\alpha_s(M_Z)) (HPQCD)</th>
<th>(\alpha_s(M_Z)) (CSSM)</th>
<th>(\delta_{D=4})</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\log(W_{11}))</td>
<td>0.1185(8)</td>
<td>0.1190(11)</td>
<td>0.7%</td>
</tr>
<tr>
<td>(\log(W_{12}))</td>
<td>0.1185(8)</td>
<td>0.1191(11)</td>
<td>2.0%</td>
</tr>
<tr>
<td>(\log\left(\frac{W_{12}^{12}}{u_0}\right))</td>
<td>0.1183(7)</td>
<td>0.1191(11)</td>
<td>5.2%</td>
</tr>
<tr>
<td>(\log\left(\frac{W_{11}W_{22}}{W_{12}^2}\right))</td>
<td>0.1185(9)</td>
<td>N/A</td>
<td>32%</td>
</tr>
<tr>
<td>(\log\left(\frac{W_{23}}{u_0}\right))</td>
<td>0.1176(9)</td>
<td>N/A</td>
<td>53%</td>
</tr>
<tr>
<td>(\log\left(\frac{W_{14}}{W_{23}}\right))</td>
<td>0.1171(11)</td>
<td>N/A</td>
<td>79%</td>
</tr>
<tr>
<td>(\log\left(\frac{W_{11}W_{23}}{W_{12}W_{13}}\right))</td>
<td>0.1174(9)</td>
<td>N/A</td>
<td>92%</td>
</tr>
</tbody>
</table>
THE HADRONIC $\tau$ DETERMINATION

- From $\Pi_{T;ud}^{(0+1)}$, $T = V, A, V + A$ FESRs [more below]

$$\int_{0}^{s_0} w(s) \rho_{T;ud}^{(0+1)}(s) ds = -\frac{1}{2\pi i} \oint_{|s|=s_0} w(s) \Pi_{T;ud}^{(0+1)}(s) ds$$

- valid for any $s_0$, analytic $w(s)$

- LHS: data; RHS: OPE (hence $\alpha_s$) for $s_0 >> \Lambda_{QCD}^2$
The flavor $ij$ current-current 2-point functions

- With $J^\mu = V^\mu_{ij}$ or $A_i^\mu$, $\Pi_{V/A;ij}^{\mu\nu}$, and $J = 0, 1$ scalar components, $\Pi_{V/A;ij}^{(0,1)}$, defined by

\[\Pi_{V/A;ij}^{\mu\nu}(q^2) \equiv i \int d^4x e^{iq\cdot x}\langle0|T\left(J^\mu(x)J^\dagger_\nu(0)\right)|0\rangle = (q^\mu q^\nu - q^2 g^{\mu\nu}) \Pi^{(1)}_{V/A;ij}(q^2) + q^\mu q^\nu \Pi^{(0)}_{V/A;ij}(q^2)\]

- “Spectral functions”: $\rho_{V/A;ij}^{(J)} = \frac{1}{\pi} Im \Pi_{V/A;ij}^{(J)}$

- $\Pi^{(0)}_{V/A;ij}$, $\Pi^{(1)}_{V/A;ij}$: physical singularities PLUS $q^2 = 0$ “kinematic singularities” [but no kinematic singularities in $\Pi^{(0)} + \Pi^{(1)} \equiv \Pi^{(0+1)}$, $q^2 \Pi^{(0)}(q^2)$ (easiest to see using ChPT)]
- Experimental determination of $\rho_{V/A}^{(0+1)}_{ud}$

- With $R_{V/A;ij} = \frac{\Gamma[\tau \rightarrow \nu_{\tau} \text{hadrons}_{V/A;ij}(\gamma)]}{\Gamma[\tau \rightarrow \nu_{\tau} e^{-\bar{\nu}_{e}}(\gamma)]}$, $y_{\tau} = s/m_{\tau}^{2}$,

  $$w_{T}(y) = (1 - y)^{2}(1 + 2y), \quad w_{L}(y) = -2y(1 - y)^{2},$$

  $$R_{V/A;ij} = 12\pi^{2}|V_{ij}|^{2} S_{EW} \int_{0}^{1} dy_{\tau} \left[ w_{T}(y_{\tau}) \rho_{V/A;ij}^{(0+1)}(s) ight. + \left. w_{L}(y_{\tau}) \rho_{V/A;ij}^{(0)}(s) \right]$$

- In QCD, $\rho_{A;ud}^{(0)}(s) = 2f_{\pi}^{2}\delta(s - m_{\pi}^{2}) + O[(m_{d} + m_{u})^{2}]$,

  $\rho_{V;ud}^{(0)}(s) = O[(m_{d} + m_{u})^{2}]$

  $\Rightarrow \rho_{V/A;ud}^{(1)}(s)$ from continuum $dR_{V/A;ud}/ds$
• The V, A and V+A spectral integrals

- V+A, \( I = 1 \) spectral function \( \rho_{V+A;ud}^{(0+1)}(s) \) from experimental differential decay distribution \( \frac{dR_{V+A;ud}}{ds} \)

- V, A separation: for \( n \pi \) states: V if \( n \) even, A if \( n \) odd; \( K\bar{K} \): pure V ⇒ separation unambiguous except for \( K\bar{K}\pi, K\bar{K}\pi\pi, \cdots \)

- ⇒ \((J) = (0 + 1); w(s)\)-weighted, \( 0 < s \leq s_0 \leq m_T^2 \), V, A or V+A spectral integrals from \( dR_{V/A;ud}/ds \)

\[
I_{spe;T}^w(s_0) = \int_0^{s_0} ds \, w(s) \rho_{T;ud}^{(0+1)}(s)
\]
• The OPE FESR integrals:

- $D = 0$: fixed by $\alpha_s$ (known to 5 loops); strongly dominant for $s_0 \gtrsim 2 \text{ GeV}^2$ [CIPT/FOPT]

- $D = 2$: $\propto (m_d \pm m_u)^2$, hence negligible

- $D = 4$: fixed by $\langle aG^2 \rangle$, $\langle m_\ell \bar{\ell} \ell \rangle$, $\langle m_s \bar{s}s \rangle$

- $D = 6, 8, \ldots$:
  * not known phenomenologically, hence fitted to data (or guesstimated)

  * for $\sim 1\%$ $\alpha_s(M_Z)$ determination need integrated $D > 4$ to $\lesssim 0.5\%$ of $D = 0$
More on fitting the $D > 4$ contributions

* $w(y) = \sum_{m=0} b_m y^m$, $y = s/s_0$ to distinguish contribs with different $D$ (differing $s_0$ dependence)

* integrated $D = 2k + 2 \geq 2$ contribution $\Leftrightarrow b_k \neq 0$ (up to $O[\alpha_s^2(m_T^2)]$ corrections)

$\Rightarrow$ OPE to $D_{max} = 2N + 2$ for degree $N$ $w(y)$

* integrated $D = 2k + 2$ contributions ($\propto 1/s_0^k$)

$$\frac{-1}{2\pi i} \oint_{|s| = s_0} ds \, w(y) \sum_{D>4} \frac{C_D}{Q_D} = \sum_{k \geq 2} (-1)^k b_k \frac{C_{2k+2}}{s_0^k}$$
Summary of recent $\tau$-based determinations

- Differences in 6-loop $D = 0$ Adler function coeff, $d_5$; $D = 0$ series integral prescription; $D > 4$ treatment

- Duality violation typically assumed negligible

<table>
<thead>
<tr>
<th>Source</th>
<th>$d_5$</th>
<th>$D &gt; 4$ self-consistency</th>
<th>PT scheme</th>
<th>$\alpha_s(M_Z^2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>BCK08</td>
<td>275</td>
<td>No</td>
<td>$\frac{1}{2}(\text{FO}+\text{CI})$</td>
<td>0.1202(19)</td>
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<tr>
<td>ALEPH08</td>
<td>383</td>
<td>No</td>
<td>$\text{CI}$</td>
<td>0.1211(11)</td>
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<tr>
<td>BJ08</td>
<td>283</td>
<td>No</td>
<td>$\text{FO}$</td>
<td>0.1185(14)</td>
</tr>
<tr>
<td></td>
<td>283</td>
<td>No</td>
<td>$\text{model}$</td>
<td>0.1179(8)</td>
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<tr>
<td>MY08</td>
<td>275</td>
<td>Yes</td>
<td>$\text{CI}$</td>
<td>0.1187(16)</td>
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<tr>
<td>N09</td>
<td>0</td>
<td>partly</td>
<td>$\frac{1}{2}(\text{FO}+\text{CI})$</td>
<td>0.1192(10)</td>
</tr>
<tr>
<td>M09</td>
<td>400</td>
<td>No</td>
<td>$\frac{1}{2}(\text{RC}+\text{CI})$</td>
<td>0.1213(11)</td>
</tr>
<tr>
<td>CF09</td>
<td>283</td>
<td>No</td>
<td>modified CI</td>
<td>0.1186(13)</td>
</tr>
</tbody>
</table>
THE ALEPH, OPAL (AND RELATED) ANALYSES

- \( w_{(00)}(y) = 1 - 3y^2 + 2y^3 \Rightarrow \text{OPE up to } D = 6, 8 \)

- \( \Gamma[\tau \rightarrow \text{hadrons}_{ud} \nu_\tau] \) alone (\( \leftrightarrow I_{\text{spec,} V + A(m_\tau^2)}^{w(00)} \)) insufficient to fix \( \alpha_s, C_6, C_8 \)

- ALEPH, OPAL approach
  - add \( s_0 = m_\tau^2, (km) = (10), (11), (12), (13) \) “spectral weight” FESRs \( [w(y) \rightarrow y^m (1 - y)^k w_{(00)}(y)] \)
  - neglect (in ppl present) \( D = 10, \cdots, 16 \) contribs
  - \( \alpha_s, \langle aG^2 \rangle, C_6, C_8 \) fitted to 5 integral set
• **NOTE:** ALEPH $C_6, C_8$ input to most other analyses

• Potential problem: single $s_0 (= m_T^2) \Rightarrow D > 8$ (if non-negligible) distort $D = 0, 4, 6, 8$ fit parameters

• Test for possible symptoms (systematic $s_0$-dependence problems) using “fit qualities”

\[
F_T^w(s_0) \equiv \left[ I_{\text{spec}; T}(s_0) - I_{\text{OPE}; T}(s_0) \right] / \delta I_{\text{spec}; T}(s_0)
\]

• **FIGURE:** $F_V^w(s_0)$ for ALEPH data, OPE fit, and 3 $w(k,m)$ used in ALEPH/OPAL fit, PLUS 3 other degree 3 $w(y)$ (to provide independent $C_6,8$ tests)
- OPE-spectral mismatch $\Rightarrow$ either a problem with assumption that $D > 8$ negligible, or OPE breakdown (either way a problem for extracted $\alpha_s$)
A MODIFIED ANALYSIS

- $V, A$ and $V+A$, $w_N(y) \equiv 1 - \frac{N}{N-1}y + \frac{1}{N-1}y^N$ FESRs


- single unsuppressed $D = 2N + 2 > 4$ contrib ($N \geq 2$),
  
  $(-1)^N C_{2N+2} / [(N-1)s_0^N]$

- $1/s_0^{N+1}$ scaling c.f. $D = 0 \Rightarrow$ joint $\alpha_s, C_{2N+2}$ fit

- $1/(N-1)$ $D = 2N + 2$ suppression, no $D = 0$ suppression $\Rightarrow$ MUCH better $\alpha_s$ emphasis than $w_{(k,m)}$ set
RESULTS

- Results for $\alpha_s(m_T^2)$ using the CIPT $D = 0$ prescription

<table>
<thead>
<tr>
<th>$w(y)$</th>
<th>ALEPH V+A</th>
<th>OPAL V+A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>0.320(5)(12)</td>
<td>0.322(7)(12)</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.320(5)(12)</td>
<td>0.322(7)(12)</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.320(5)(12)</td>
<td>0.322(7)(12)</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.320(5)(12)</td>
<td>0.322(7)(12)</td>
</tr>
<tr>
<td>$w_6$</td>
<td>0.320(5)(12)</td>
<td>0.322(8)(12)</td>
</tr>
</tbody>
</table>

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<thead>
<tr>
<th>$w(y)$</th>
<th>ALEPH V</th>
<th>ALEPH A</th>
<th>ALEPH V+A</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>0.321(7)(12)</td>
<td>0.319(6)(12)</td>
<td>0.320(5)(12)</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.321(7)(12)</td>
<td>0.319(6)(12)</td>
<td>0.320(5)(12)</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.321(7)(12)</td>
<td>0.319(6)(12)</td>
<td>0.320(5)(12)</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.321(7)(12)</td>
<td>0.319(6)(12)</td>
<td>0.320(5)(12)</td>
</tr>
<tr>
<td>$w_6$</td>
<td>0.321(7)(12)</td>
<td>0.319(6)(12)</td>
<td>0.320(5)(12)</td>
</tr>
</tbody>
</table>
• Much improved $F_V^{w}(s_0)$ for $w = w_N$ c.f. $w = w_{(k,m)}$
• CIPT $w_2, \cdots, w_6$ fit values consistent to $\pm 0.0001$

• Averaging ALEPH and OPAL based results with non-normalization component of error $\Rightarrow$

$$\alpha_s^{(n_f=3)}(m_\tau) = 0.3209(46)_{exp}(118)_{th}$$

• standard self-consistent combination of 4-loop running, 3-loop matching at flavor thresholds $\Rightarrow$

$$\alpha_s^{(n_f=5)}(M_Z) = 0.1187(3)_{evol}(6)_{exp}(15)_{th}$$
CONCLUSIONS/SUMMARY/PROSPECTS

- Lattice \( \log(W_{11}) \) to be specific) and \( \tau \) determinations now in excellent agreement

\[
[\alpha_s(M_Z)]_{\text{latt}} = 0.1185(8), \ 0.1190(11)
\]

\[
[\alpha_s(M_Z)]_{\tau} = 0.1187(16)
\]

- Future prospects:
  - Significant improvement to lattice errors difficult
  - Some improvement in \( \tau \) decay analysis probable
MORE ON FUTURE PROSPECTS

- The lattice analysis case:
  - further self-consistency checks from additional $a \sim 0.045$ fm MILC ensembles, BUT $a$ small enough to avoid fitting HO $D = 0$ coefficients impractical

\[
\begin{align*}
\alpha_s(M_Z^2) \text{ with only known vs with fitted HO coefficients}
\end{align*}
\]
– dominant overall scale-setting error, residual HO
\[ D = 0 \] issues hence difficult to improve significantly

• The \( \tau \) decay analysis case:

  Significant improvement requires better understanding of \( D = 0 \) truncation uncertainty and residual duality violation (if any)

  – Theory error currently dominant (\( \sim 2.5 \) times expt’l)

  – \( D = 0 \) truncation is largest theory error source (for \( |FOPT - CIPT| \pm O(a^5) \left[ \delta \alpha_s(m_{\tau}^2) \right]_{triunc} \) estimate, \( \sim 0.010 \) of 0.012 theory total) \( \Rightarrow \) important bottleneck for future improvements
Future possibilities re CIPT vs. FOPT for $D = 0$:

* Weight choice to reduce FOPT/CIPT difference

* Recent explorations a la Beneke-Jamin, Caprini-Fischer (taking into account divergent nature of $D = 0$ series, resummation)

* Renormalon ambiguities for $D = 4, 6, \cdots$ effective condensates from IR renormalons in resummed $D = 0$ series also imply $D = 0$ truncation, $D = 4, 6, \cdots$ fitting, tied in with residual duality violation question

* $\Rightarrow$ investigations of duality violation also needed
— Constraining duality violation, and related issues:

* DV certainly present (e.g., in spectral functions)

* Plausible Regge-inspired model for DV effects [Cata, Goltermann, Peris (CGP)]

* Fitting model to spectral function alone insufficient (poor theory to experiment FESR matches for fitted CGP parameters)

* Further FESR constraints on CGP DV model now demonstrated [KM+CGP+Jamin...], with simultaneous $\langle aG^2 \rangle$ determination
* \( \langle aG^2 \rangle \) determination relevant because
  
  - \( \langle aG^2 \rangle \) determination NOT feasible for \( w(y) \) with DV suppressed enough to be neglected

  - Input \( \langle aG^2 \rangle \) second largest MY08 theory error

  - \( \langle aG^2 \rangle \) renormalon ambiguity \( \Rightarrow \) should determine simultaneously with truncated \( D = 0 \) series

* Preliminary results show additional constraints push maximum DV impact on \( \alpha_s \) determination to low end of former CGP range (\( \sim \) 0.0004 on \( \alpha_s(M_Z) \))

  - Non-trivial \( \tau \) determination error reduction thus appears feasible, though not yet explicitly demonstrated
PRELIMINARY RESULTS ON DV

- *Duality violation certainly present (e.g. $\rho_{ud}; V(s)$):*
• $ud \ V, \ w(y)=1$ FESR with central CGP DV parameters
- $ud V, w(y) = 1$ FESR with FESR-optimized DV parameters
SUPPLEMENTARY $\tau$ MATERIAL

- More on consistency of V+A fit results

- More on the independence of the $w_2, \cdots, w_6$ FESRs

- Some observations on the Beneke-Jamin calculation
More on the consistency of the V+A fit results

V+A fit results for $\alpha_s(m_\tau)$

<table>
<thead>
<tr>
<th>$w(y)$</th>
<th>CIPT full fit</th>
<th>$s_0 = m_\tau^2$ CIPT $D &gt; 4 \rightarrow 0$</th>
<th>FOPT full fit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>0.320</td>
<td>0.310</td>
<td>0.320</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.320</td>
<td>0.316</td>
<td>0.315</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.320</td>
<td>0.319</td>
<td>0.313</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.320</td>
<td>0.321</td>
<td>0.312</td>
</tr>
<tr>
<td>$w_6$</td>
<td>0.320</td>
<td>0.322</td>
<td>0.312</td>
</tr>
</tbody>
</table>
More on the independence of the $w_2, \ldots, w_6$ FESRs

Fitted ALEPH-based $V+A$ $\alpha_s(m_T^2)$ from pseudo-FESRs employing one $w_N$ for the spectral integrals (row label) and another for the OPE integrals (column heading)

<table>
<thead>
<tr>
<th></th>
<th>$w_2$</th>
<th>$w_3$</th>
<th>$w_4$</th>
<th>$w_5$</th>
<th>$w_6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_2$</td>
<td>0.320</td>
<td>0.175</td>
<td>—</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>$w_3$</td>
<td>0.435</td>
<td>0.320</td>
<td>0.249</td>
<td>0.194</td>
<td>0.149</td>
</tr>
<tr>
<td>$w_4$</td>
<td>0.499</td>
<td>0.384</td>
<td>0.320</td>
<td>0.277</td>
<td>0.243</td>
</tr>
<tr>
<td>$w_5$</td>
<td>0.541</td>
<td>0.423</td>
<td>0.361</td>
<td>0.320</td>
<td>0.291</td>
</tr>
<tr>
<td>$w_6$</td>
<td>—</td>
<td>0.450</td>
<td>0.388</td>
<td>0.349</td>
<td>0.320</td>
</tr>
</tbody>
</table>
Some observations on the Beneke-Jamin calculation

- As for the spectral weight analysis, control of $D > 4$ contributions essential for precision $\alpha_s$ (independent of choice of FOPT or CIPT for $D = 0$ contributions)

- Can test BJ input assumptions for $C_{6,8}$ for consistency with output FOPT fit $\alpha_s$ using $F^{w}_{V+A}(s_0)$ for various degree $\leq 3$ $w(y)$ (FIGURE)

- Find problems for combination of assumed $D = 6, 8$ and FOPT fitted $\alpha_s$
• Exercise to test implications of (minimal, 5-parameter) BJ model for the resummed $D = 0$ series

  – Features of the minimal model:

    * good approximation to full model sum using FOPT for a range of $w(y)$ (FIGURES)

    * CIPT approximation inferior to FOPT most strongly so for $w_{(0,0)}$ (FIGURES)

    * ⇒ expect consistency of various FOPT fits, reduced consistency for CIPT fits

  – FIGURE: FOPT, CIPT vs. Borel sum for BJ model
$w(0,0)$

$w_2$

Maltman weight $w_2$
- Test expectations with combined FOPT, CIPT \(w_2\)-\(w_3\) fit

  * combined fit yields \(\alpha_s, C_6, C_8\), hence OPE integrals fixed for any degree \(\leq 3\) \(w(y)\)

  * test agreement of CIPT, FOPT OPE with corresponding spectral integrals for \(w_{(0,0)}, y(1-y)^2\)

- find good (not good) CIPT (FOPT) consistency (contrary to model expectations) (FIGURE)

- suggests alternate non-minimal modelling possible using such observations as constraints
FOPT vs CIPT $w_2 - w_3$ joint fit V+A fit qualities