The $\Delta I = 1/2$ Rule: An Old Enigma

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The Problem

- Strangeness changing sector of the Standard Model.
- $|\Delta S|=1$, nonleptonic weak decays of mesons and hyperons.
- Nature favours $|\Delta I|=1/2$ transitions
  $\Rightarrow \Delta I=1/2$ selection rule.
- Explanation still elusive.
The Problem

• The received wisdom is that the solution lies in the low-energy dynamics of QCD.
• We considered a class of electroweak radiative corrections to the $K\pi\pi$ effective vertex.
• Hoped to find a significant $\Delta I=1/2$ effect and provide a check.
Nonleptonic Weak Interactions

- Model weak interactions as \{current-current\}

\[
\mathcal{L}^W = \frac{g^2}{8} \int W^{\mu\nu}(x, m_W) T \left\{ J^+_{\mu}(x/2) J_\nu(-x/2) \right\}
\]

- Complicated structure \(\Rightarrow\) difficult to analyse.
- Need control over low, intermediate, and high energies to make predictions.
Nonleptonic Weak Interactions

- $|\Delta S|=1$ decays governed by currents:

$$ (\bar{d}_L \gamma^\mu u_L) (\bar{u}_L \gamma_\mu s_L); \quad (\bar{d}_L \gamma^\mu c_L) (\bar{c}_L \gamma_\mu s_L); \quad (\bar{d}_L \gamma^\mu t_L) (\bar{t}_L \gamma_\mu s_L) $$

- First product transforms under octet (adjoint) and 27-plet representations of SU(3).

- From symmetry considerations can write chiral effective Lagrangian.
Chiral Lagrangians

- $\chi$PT useful tool for studying meson-dynamics.
- Treat $(\pi, K, \eta)$ as effective degrees of freedom and encode in $3 \times 3$ matrix field $U$:

$$U^\dagger U = I; \quad \det U = 1$$

- Provided effective potential minimised then:

$$U = e^{-i\Phi/F_0}; \quad \Phi := \begin{pmatrix} -\pi^0 + \sqrt{\frac{1}{3}}\eta & \sqrt{2}\pi^+ & \sqrt{2}K^+ \\ \sqrt{2}\pi^- & -\pi^0 + \sqrt{\frac{1}{3}}\eta & \sqrt{2}K^0 \\ \sqrt{2}K^- & \sqrt{2}K^0 & -2\sqrt{\frac{1}{3}}\eta \end{pmatrix}$$
• In $\Delta S=0$ sector use lowest order Lagrangian to model strong interactions.

• Weak interactions given by effective chiral Lagrangian:

\[ \mathcal{L}_{\text{eff}}^{W} = \sum_{i} g_{i} \mathcal{L}_{i} \left[ U \partial_{\mu} U^{\dagger} \right] + \Delta[U] + \mathcal{O}(\chi^2) + \text{h.c.} \]

• Use symmetries to eliminate as many terms as possible.
Chiral Lagrangians

• Zero-trace condition $\text{Tr} \left[ U \partial_\mu U^\dagger \right] = 0$ yields octet contribution to $\Delta S=1$ transitions:

$$\mathcal{L}_8 = (U \partial_\mu U^\dagger)_{13} (U \partial_\mu U^\dagger)_{21} - (U \partial_\mu U^\dagger)_{23} (U \partial_\mu U^\dagger)_{11}$$

• Subtle point: $\Delta[U]$ does not contribute to physical processes.

• Thus $\mathcal{L}_8$ responsible for pure $\Delta l=1/2$ transitions (octet dominance) in K decays.
Chiral Lagrangians

• From 27-plet get both $\Delta I=1/2$ and $\Delta I=3/2$ contributions.

• Deduce contributions via U-spin:

$$\mathcal{L}_{27} = \left( U \partial_\mu U^\dagger \right)_{13} \left( U \partial^\mu U^\dagger \right)_{21} + \frac{3}{2} \left( U \partial_\mu U^\dagger \right)_{23} \left( U \partial^\mu U^\dagger \right)_{11}$$

• Thus get: $$\mathcal{L}_{\text{eff}}^W = g_8 \mathcal{L}_8 + g_{27} \mathcal{L}_{27} + \text{h.c.}$$
K→2\(\pi\) Decay

- Bose symmetry ⇒ \(\pi\pi\) final state has \(I=0,2\)
- Decompose final state into isospin basis:

\[
\mathcal{A}_{K^0\to\pi^+\pi^-} = A_0 e^{i\delta_0} + \frac{A_2}{\sqrt{2}} e^{i\delta_2}
\]
\[
\mathcal{A}_{K^0\to\pi^0\pi^0} = A_0 e^{i\delta_0} - \sqrt{2} A_2 e^{i\delta_2}
\]
\[
\mathcal{A}_{K^+\to\pi^+\pi^0} = \frac{3}{2} A'_2 e^{i\delta_2}
\]

- Experiment gives: \(|A_0/A_2| \approx 22\)
K→2π Decay

• How do chiral Lagrangians fare?
• From $\mathcal{L}_8$ and $\mathcal{L}_{27}$ one finds at tree level:

$$A_0 = \frac{\sqrt{2}g_8}{F^3_\pi} \left( m_K^2 - m_\pi^2 \right); \quad A_2 = \frac{2g_{27}}{F^3_\pi} \left( m_K^2 - m_\pi^2 \right)$$

• But $g_8$ and $g_{27}$ not fixed *a priori* by chiral symmetry alone.
• Explaining $\Delta I=1/2$ rule $\iff$ explaining origin of effective couplings.
Short Distance Effects

• Idea: Use asymptotic freedom property of QCD to use perturbation theory for energy range $m_W \geq E \geq \mu$.

• Via operator product expansion can obtain effective Lagrangian:

$$\mathcal{L}^{\Delta S=1} = \frac{G_F}{2\sqrt{2}} V_{ud}^* V_{us} \sum_i c_i(\mu) \mathcal{O}_i$$
Short Distance Effects

• $\mathcal{L}_{\Delta S=1}$ generates diagrams of form:

• Strong radiative corrections generate new operators.

• $\Delta I=1/2$ enhancement of $\sim 2-5$. 
Electroweak Corrections

• Interested in corrections to $K\pi\pi$ effective vertex:
Electroweak Corrections

- The $s d \gamma$ vertex consists of:
The Rise of the Dilaton?

• Dilaton is Goldstone boson associated with breaking of scale invariance: \( x^\mu \rightarrow \lambda x^\mu \)

• Received wisdom attributes broken scale invariance to trace anomaly.

• What happens at infrared fixed point of QCD?

• Potential contributions from dilaton to \( K^0 \) decays.