Variations in Binding Energies with respect to Changes in Fundamental Constants

Manuel E. Carrillo-Serrano

Centre for the Subatomic Structure of Matter (CSSM)
The University of Adelaide

November 23, 2011
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with \( m_q \)

Results

Outline

1 Motivation
   - Experimental Evidence
   - Time shifts in \( \alpha \) and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3 Variations in the mass of the Mesons with \( m_q \)
   - \( \pi \)-meson
   - \( \sigma \)-meson
   - \( \rho \)-meson and \( \omega \)-meson

4 Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for \( ^7\text{Li}, ^{12}\text{C}, ^{16}\text{O} \) Nuclei
Motivation

Astrophysical Data has shown that the fine structure constant $\alpha$ changes in time and space.\(^1\)


Motivation

Astrophysical Data has shown that the fine structure constant $\alpha$ changes in time and space.\(^1\)

Withing Grand Unified Theories (GUTS), this implies that other quantities would also change with $\alpha$: $\alpha_{\text{weak}}$, $\alpha_{\text{strong}}$, $\Lambda_{\text{QCD}}$, and $m_q$.\(^2\)

---


Motivation

1. Astrophysical Data has shown that the fine structure constant $\alpha$ changes in time and space.\textsuperscript{1}

2. Withing Grand Unified Theories (GUTS), this implies that other quantities would also change with $\alpha$: $\alpha_{\text{weak}}$ and $\alpha_{\text{strong}}$, $\Lambda_{\text{QCD}}$, and $m_q$.\textsuperscript{2}

3. If GUTS are correct, variations in $m_q$ would imply variations in the masses of different mesons.

---

\textsuperscript{1}J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Lett. 82, 884 (1999).

Motivation

1. Astrophysical Data has shown that the fine structure constant $\alpha$ changes in time and space.\(^1\)

2. Within Grand Unified Theories (GUTS), this implies that other quantities would also change with $\alpha$: $\alpha_{\text{weak}}$ and $\alpha_{\text{strong}}$, $\Lambda_{\text{QCD}}$, and $m_q$.\(^2\)

3. If GUTS are correct, variations in $m_q$ would imply variations in the masses of different mesons.

4. The masses of the exchanged mesons in One Boson Exchange potentials (OBE) would also change and therefore Binding Energies.

---


Motivation

1. Astrophysical Data has shown that the fine structure constant $\alpha$ changes in time and space.\(^1\)

2. Within Grand Unified Theories (GUTS), this implies that other quantities would also change with $\alpha$: $\alpha_{\text{weak}}$ and $\alpha_{\text{strong}}$, $\Lambda_{\text{QCD}}$, and $m_q$.\(^2\)

3. If GUTS are correct, variations in $m_q$ would imply variations in the masses of different mesons.

4. The masses of the exchanged mesons in One Boson Exchange potentials (OBE) would also change and therefore Binding Energies.

---


1 Motivation
   - Experimental Evidence
     - Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4 Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
The energy equation for the transition from a ground state to a multiplet state at some redshift $z$ is:\(^3\)

The energy equation for the transition from a ground state to a multiplet state at some redshift $z$ is:\(^3\)

\[
E_z = E_{z=0} + \left[ Q_1 + K_1 (LS) \right] Z^2 \left[ \left( \frac{\alpha_z}{\alpha_0} \right)^2 - 1 \right]
+ K_2 (LS)^2 Z^4 \left[ \left( \frac{\alpha_z}{\alpha_0} \right)^4 - 1 \right].
\]

With $Q_1$ the relativistic correction to $E_{z=0}$, $K_1$, and $K_2$ the relativistic coefficients to the spin-orbit contributions, $Z$ the nuclear charge, and $\alpha_0$ is the value of $\alpha$ at $z = 0$.

Experimental Evidence

The energy equation for the transition from a ground state to a multiplet state at some redshift $z$ is:

$$E_z = E_{z=0} + [Q_1 + K_1 (LS)] Z^2 \left[ \left( \frac{\alpha_z}{\alpha_0} \right)^2 - 1 \right]$$

$$+ K_2 (LS)^2 Z^4 \left[ \left( \frac{\alpha_z}{\alpha_0} \right)^4 - 1 \right].$$

With $Q_1$ the relativistic correction to $E_{z=0}$, $K_1$, and $K_2$ the relativistic coefficients to the spin-orbit contributions, $Z$ the nuclear charge, and $\alpha_0$ is the value of $\alpha$ at $z = 0$.

---


---
For Fe II transition frequencies with $\alpha$ are dominated by $Q_1$, whereas for Mg II transitions are small due to small $Z$. 
For Fe II transition frequencies with $\alpha$ are dominated by $Q_1$, whereas for Mg II transitions are small due to small $Z$.

High quality data from Quasar absorption spectra show at low/intermediate redshift Fe II and Mg II transitions (and other species). The relative positions of the lines in Fe II and Mg II were measured in order to find $\frac{\Delta \alpha}{\alpha}$. 
For Fe II transition frequencies with $\alpha$ are dominated by $Q_1$, whereas for Mg II transitions are small due to small $Z$.

High quality data from Quasar absorption spectra show at low/intermediate redshift Fe II and Mg II transitions (and other species), The relative positions of the lines in Fe II and Mg II were measured in order to find $\frac{\Delta \alpha}{\alpha}$. 
Experimental Evidence

4

\textsuperscript{4} Taken from: J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Lett. 82, 884 (1999).
Experimental Evidence

\[ \frac{\Delta \omega}{\omega} \]

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{plot.png}
\caption{Experimental Evidence}
\end{figure}

\[ \text{Redshift} \]

\[ 0.6 \quad 0.8 \quad 1.0 \quad 1.2 \quad 1.4 \quad 1.6 \]

\[ 0 \quad 0.0001 \quad 0.001 \quad 0.01 \]

\[^{4}\text{Taken from: J.K. Webb, V.V. Flambaum, C.W. Churchill, M.J. Drinkwater, and J.D. Barrow, Phys. Lett. 82, 884 (1999).} \]
Experimental Evidence

- Total variation of $10^{-5}$ in 30 Fe II/Mg II absorption systems.

Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

Experimental Evidence

Time shifts in $\alpha$ and its implications on the nucleon mass

Outline

1 Motivation
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4 Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

Experimental Evidence
Time shifts in $\alpha$ and its implications on the nucleon mass

Time shifts in $\alpha$ and its implications on the nucleon mass

- The three coupling constants come together at energies of $10^{16}$ GeV.
The three coupling constants come together at energies of $10^{16}$ GeV.

Assumption: Small shift of $\alpha$ imply changes in all three coupling constants. If not convergence will happen at specific time.
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Experimental Evidence
Time shifts in $\alpha$ and its implications on the nucleon mass

Time shifts in $\alpha$ and its implications on the nucleon mass

- The three coupling constants come together at energies of $10^{16}$ GeV.
- Assumption: Small shift of $\alpha$ imply changes in all three coupling constants. If not convergence will happen at specific time.
- From the asymptotic freedom result at first order:

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \left( \frac{\Lambda^2_{QCD}}{\mu^2} \right)}$$  (1)
Time shifts in $\alpha$ and its implications on the nucleon mass

- The three coupling constants come together at energies of $10^{16}$ GeV.
- Assumption: Small shift of $\alpha$ imply changes in all three coupling constants. If not convergence will happen at specific time.
- From the asymptotic freedom result at first order:

$$\alpha_s(\mu^2) = \frac{4\pi}{\beta_0 \ln \left( \frac{\Lambda_{QCD}^2}{\mu^2} \right)}$$  \hspace{1cm} (1)

Calmet and Fritzsch found a relative variation of $\Lambda_{QCD} = 213$ MeV, in terms of $\alpha$ (Fine structure constant). At unification scale $10^{16}$ GeV, then:
Time shifts in $\alpha$ and its implications on the nucleon mass

- The three coupling constants come together at energies of $10^{16}$ GeV.
- Assumption: Small shift of $\alpha$ imply changes in all three coupling constants. If not convergence will happen at specific time.
- From the asymptotic freedom result at first order:

$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0 \ln \left( \frac{\Lambda_{QCD}^2}{\mu^2} \right)}$$  \hspace{1cm} (1)$$

Calmet and Fritzsch found a relative variation of $\Lambda_{QCD}$ 213 MeV, in terms of $\alpha$ (Fine structure constant). At unification scale $10^{16}$ GeV, then:

$$\frac{\delta\Lambda_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\delta\alpha_s}{\alpha_s}$$  \hspace{1cm} (2)$$
Time shifts in $\alpha$ and its implications on the nucleon mass

- The three coupling constants come together at energies of $10^{16}$ GeV.
- Assumption: Small shift of $\alpha$ imply changes in all three coupling constants. If not convergence will happen at specific time.
- From the asymptotic freedom result at first order:

$$\alpha_S(\mu^2) = \frac{4\pi}{\beta_0 \ln\left(\frac{\Lambda^2_{QCD}}{\mu^2}\right)}$$ (1)

Calmet and Fritzsch found a relative variation of $\Lambda_{QCD}$ 213 MeV, in terms of $\alpha$ (Fine structure constant). At unification scale $10^{16}$ GeV, then:

$$\frac{\delta \Lambda_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\delta \alpha_s}{\alpha_s}$$ (2)
If $\alpha_s$ shifts in time, therefore $M_{\text{Nucleon}}$ and $M_{\text{Hadrons}}$ would change in proportion to $\Lambda_{\text{QCD}}$. 

---

If $\alpha_s$ shifts in time, therefore $M_{Nucleon}$ and $M_{Hadrons}$ would change in proportion to $\Lambda_{QCD}$.

GUTS consider convergence of coupling constants.

---

If $\alpha_S$ shifts in time, therefore $M_{\text{Nucleon}}$ and $M_{\text{Hadrons}}$ would change in proportion to $\Lambda_{QCD}$.

- GUTS consider convergence of coupling constants.
- Calculation in SU(5) broken to the gauge group of the minimal supersymmetric extension of the Standard Model (MSSM). In 1-loop approximation:

\[ \text{X. Calmet, H. Fritzsch, arXiv:hep-ph/0112110} \]
Time shifts in $\alpha$ and its implications on the nucleon mass

- If $\alpha_S$ shifts in time, therefore $M_{\text{Nucleon}}$ and $M_{\text{Hadrons}}$ would change in proportion to $\Lambda_{\text{QCD}}$.
- GUTS consider convergence of coupling constants.
- Calculation in SU(5) broken to the gauge group of the minimal supersymmetric extension of the Standard Model (MSSM). In 1-loop approximation:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i^0(\mu^0)} + \frac{1}{2\pi} b_i \ln \left( \frac{\mu_0}{\mu} \right) \quad (3)$$

\[5\]

Time shifts in $\alpha$ and its implications on the nucleon mass

- If $\alpha_S$ shifts in time, therefore $M_{Nucleon}$ and $M_{Hadrons}$ would change in proportion to $\Lambda_{QCD}$.
- GUTS consider convergence of coupling constants.
- Calculation in SU(5) broken to the gauge group of the minimal supersymmetric extension of the Standard Model (MSSM). In 1-loop approximation:

$$\frac{1}{\alpha_i(\mu)} = \frac{1}{\alpha_i^0(\mu^0)} + \frac{1}{2\pi} b_i \ln \left( \frac{\mu_0}{\mu} \right)$$  \hspace{1cm} (3)

\footnote{X. Calmet, H. Fritzsch, arXiv:hep-ph/0112110}
with \( b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7) \rightarrow \) below supersymmetric scale.
And \( b_i^{S} = (b_1^{S}, b_2^{S}, b_3^{S}) = (33/5, 1, -3) \rightarrow \) Just when supersymmetry is restored.
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Experimental Evidence
Time shifts in $\alpha$ and its implications on the nucleon mass

with $b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7) \rightarrow$ below
supersymmetric scale.
And $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3) \rightarrow$ Just when
supersymmetry is restored.

- If $\alpha_i (\mu, t)$, then:
with $b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7) \rightarrow$ below supersymmetric scale.

And $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3) \rightarrow$ Just when supersymmetry is restored.

If $\alpha_i(\mu, t)$, then:

$$\frac{1}{\alpha_i(\mu)} \frac{\dot{\alpha}_i(\mu)}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu')} \frac{\dot{\alpha}_i(\mu')}{\alpha_i(\mu')}$$

(4)
with \( b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7) \rightarrow \) below supersymmetric scale.
And \( b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3) \rightarrow \) Just when supersymmetry is restored.

- If \( \alpha_i(\mu, t) \), then:

\[
\frac{1}{\alpha_i(\mu)} \frac{\dot{\alpha}_i(\mu)}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu')} \frac{\dot{\alpha}_i(\mu')}{\alpha_i(\mu')} \quad (4)
\]

Scale independent.
with $b_i^{SM} = (b_1^{SM}, b_2^{SM}, b_3^{SM}) = (41/10, -19/6, -7) \rightarrow$ below supersymmetric scale.
And $b_i^S = (b_1^S, b_2^S, b_3^S) = (33/5, 1, -3) \rightarrow$ Just when supersymmetry is restored.

- If $\alpha_i(\mu, t)$, then:

$$\frac{1}{\alpha_i(\mu)} \frac{\dot{\alpha}_i(\mu)}{\alpha_i(\mu)} = \frac{1}{\alpha_i(\mu')} \frac{\dot{\alpha}_i(\mu')}{\alpha_i(\mu')}$$

(4)

Scale independent.
Scale independence together with convergence of coupling constants imply:
Scale independence together with convergence of coupling constants imply:

\[
\frac{1}{\alpha_1(\mu)} \frac{\dot{\alpha}_1(\mu)}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)} \frac{\dot{\alpha}_2(\mu)}{\alpha_2(\mu)} = \frac{1}{\alpha_s(\mu)} \frac{\dot{\alpha}_s(\mu)}{\alpha_s(\mu)}. \tag{5}
\]
Scale independence together with convergence of coupling constants imply:

\[
\frac{1}{\alpha_1(\mu)} \frac{\dot{\alpha}_1(\mu)}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)} \frac{\dot{\alpha}_2(\mu)}{\alpha_2(\mu)} = \frac{1}{\alpha_s(\mu)} \frac{\dot{\alpha}_s(\mu)}{\alpha_s(\mu)}. \tag{5}
\]

Using the Weinberg angle to relate the electromagnetic and weak coupling constants, and the previous result:
Scale independence together with convergence of coupling constants imply:

\[
\frac{1}{\alpha_1(\mu)} \frac{\dot{\alpha}_1(\mu)}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)} \frac{\dot{\alpha}_2(\mu)}{\alpha_2(\mu)} = \frac{1}{\alpha_s(\mu)} \frac{\dot{\alpha}_s(\mu)}{\alpha_s(\mu)}.
\]

(5)

Using the Weinberg angle to relate the electromagnetic and weak coupling constants, and the previous result:

\[
\frac{\Lambda_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\dot{\alpha}}{\alpha} (\mu = 0)
\]
Scale independence together with convergence of coupling constants imply:

\[
\frac{1}{\alpha_1(\mu)} \frac{\dot{\alpha}_1(\mu)}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)} \frac{\dot{\alpha}_2(\mu)}{\alpha_2(\mu)} = \frac{1}{\alpha_s(\mu)} \frac{\dot{\alpha}_s(\mu)}{\alpha_s(\mu)}.
\]  

Using the Weinberg angle to relate the electromagnetic and weak coupling constants, and the previous result:

\[
\frac{\Lambda_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\dot{\alpha}}{\alpha} (\mu = 0)
\]

Since the mass of the nucleons are proportional to \( \Lambda_{QCD} \):

\[
\frac{\dot{M}}{M} = \frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\dot{\alpha}}{\alpha}
\]
Scale independence together with convergence of coupling constants imply:

$$\frac{1}{\alpha_1(\mu)} \frac{\dot{\alpha}_1(\mu)}{\alpha_1(\mu)} = \frac{1}{\alpha_2(\mu)} \frac{\dot{\alpha}_2(\mu)}{\alpha_2(\mu)} = \frac{1}{\alpha_s(\mu)} \frac{\dot{\alpha}_s(\mu)}{\alpha_s(\mu)}.$$  \hspace{1cm} (5)

Using the Weinberg angle to relate the electromagnetic and weak coupling constants, and the previous result:

$$\frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\dot{\alpha}}{\alpha} \left( \mu = 0 \right)$$  \hspace{1cm} (6)

Since the mass of the nucleons are proportional to $\Lambda_{QCD}$:

$$\frac{\dot{M}}{M} = \frac{\dot{\Lambda}_{QCD}}{\Lambda_{QCD}} \approx 38 \frac{\dot{\alpha}}{\alpha}$$  \hspace{1cm} (7)
Outline

1. **Motivation**
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2. **The One Boson Exchange Potential.**
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3. **Variations in the mass of the Mesons with $m_q$**
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4. **Results**
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Range of the potential

\[ \pi, \sigma, \rho, \omega \]
Range of the potential

\[ \pi, \sigma, \rho, \omega \]
Simple calculation using the uncertainty principle:
Simple calculation using the uncertainty principle:

\[
\Delta E \Delta t \geq \hbar \\
\Delta t \sim \frac{\hbar}{\mu c^2} \\
\Delta r \sim c \Delta t \sim \frac{\hbar c}{\mu c^2}
\]
Range of the Potential - Yukawa result

1. Simple calculation using the uncertainty principle:

\[ \Delta E \Delta t \geq \hbar \]

\[ \Delta t \sim \frac{\hbar}{\mu c^2} \]

\[ \Delta r \sim c \Delta t \sim \frac{\hbar c}{\mu c^2} \]

2. A force of range 1 – 2 fm implies an exchanged particle with a mass of (100 – 200) \( \frac{MeV}{c} \).
Simple calculation using the uncertainty principle:

\[ \Delta E \Delta t \geq \hbar \]

\[ \Delta t \sim \frac{\hbar}{\mu c^2} \]

\[ \Delta r \sim c \Delta t \sim \frac{\hbar c}{\mu c^2} \]

A force of range 1 – 2 fm implies an exchanged particle with a mass of \((100 – 200) \frac{MeV}{c}\)
Outline

1 Motivation
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4 Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

Range of the potential

Schematic Representation

Implications on Binding Energies
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Range of the potential
Schematic Representation
Implications on Binding Energies

Schematic Representation

$$V(r)$$

Vector-meson exchange
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Range of the potential

Schematic Representation

Implications on Binding Energies

Schematic Representation

$V(r)$

1 2 3 4 5

$r$(fm)

Vector-meson exchange
Scalar-meson exchange
Schematic Representation

Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Range of the potential

Schematic Representation

Implications on Binding Energies

Vector-meson exchange  Scalar-meson exchange  One-pion exchange
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

Range of the potential

Schematic Representation

Implications on Binding Energies

Schematic Representation

$V(r)$

1

2

3

4

5

$r$(fm)

Vector-meson exchange  Scalar-meson exchange  One-pion exchange
Outline

1 Motivation
   • Experimental Evidence
   • Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   • Range of the potential
   • Schematic Representation
   • Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   • $\pi$-meson
   • $\sigma$-meson
   • $\rho$-meson and $\omega$-meson

4 Results
   • Binding energy of the Deuteron
   • Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Implications on Binding Energies

Binding energies are defined as \( M_N - A m_N \), where \( M_N \) is the mass of the nucleus and \( m_N \) is the mass of the nucleon, \( A \) is the number of nucleons.
Implications on Binding Energies

1. Binding energies are defined as $M_N - Am_N$, where $M_N$ is the mass of the nucleus and $m_N$ is the mass of the nucleon, $A$ is the number of nucleons.

2. Binding energies are due to nucleon - nucleon interactions.
Implications on Binding Energies

1. Binding energies are defined as $M_N - Am_N$, where $M_N$ is the mass of the nucleus and $m_N$ is the mass of the nucleon, $A$ is the number of nucleons.

2. Binding energies are due to nucleon - nucleon interactions.

3. Therefore OBE.
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

Implications on Binding Energies

1. Binding energies are defined as $M_N - Am_N$, where $M_N$ is the mass of the nucleus and $m_N$ is the mass of the nucleon, $A$ is the number of nucleons.

2. Binding energies are due to nucleon - nucleon interactions.

3. Therefore OBE.

4. If the mass of the mesons change therefore the OBE potential changes, and consequently Binding energies change.
Implications on Binding Energies

1. Binding energies are defined as $M_N - A m_N$, where $M_N$ is the mass of the nucleus and $m_N$ is the mass of the nucleon, $A$ is the number of nucleons.

2. Binding energies are due to nucleon - nucleon interactions.

3. Therefore OBE.

4. If the mass of the mesons change therefore the OBE potential changes, and consequently Binding energies change.
Variations in the mass of the Mesons

1. There are four mesons that we are taking into account:
Variations in the mass of the Mesons

There are four mesons that we are taking into account:

- Pions $\pi$. 
Variations in the mass of the Mesons

1. There are four mesons that we are taking into account:
   - Pions $\pi$.
   - Sigma-meson $\sigma$. 
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

Variations in the mass of the Mesons

There are four mesons that we are taking into account:

- Pions $\pi$.
- Sigma-meson $\sigma$.
- Rho-meson $\rho$. 
Variations in the mass of the Mesons

1. There are four mesons that we are taking into account:
   - Pions $\pi$.
   - Sigma-meson $\sigma$.
   - Rho-meson $\rho$.
   - Omega-meson $\omega$.
There are four mesons that we are taking into account:

- Pions $\pi$.
- Sigma-meson $\sigma$.
- Rho-meson $\rho$.
- Omega-meson $\omega$.

$\sigma$
Variations in the mass of the Mesons

1. There are four mesons that we are taking into account:
   - Pions $\pi$.
   - Sigma-meson $\sigma$.
   - Rho-meson $\rho$.
   - Omega-meson $\omega$.

2. $\sigma$

3. $\rho$ and $\omega$
Variations in the mass of the Mesons

1. There are four mesons that we are taking into account:
   - Pions $\pi$.
   - Sigma-meson $\sigma$.
   - Rho-meson $\rho$.
   - Omega-meson $\omega$.

2. $\sigma$

3. $\rho$ and $\omega$
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

π-meson
σ-meson
ρ-meson and ω-meson

Outline

1 Motivation
   • Experimental Evidence
   • Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   • Range of the potential
   • Schematic Representation
   • Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   • $\pi$-meson
   • $\sigma$-meson
   • $\rho$-meson and $\omega$-meson

4 Results
   • Binding energy of the Deuteron
   • Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$.

Results

- $\pi$-meson
- $\sigma$-meson
- $\rho$-meson and $\omega$-meson

$\pi$-meson

- Using GMOR:\(^6\)

---

Using GMOR: \(^6\)

\[
4m_q \langle 0 | \overline{q} q | 0 \rangle = -2f_\pi^2 M_\pi^2
\]  

π-meson

- Using GMOR:\(^6\)

\[
4m_q \langle 0 | \bar{q}q | 0 \rangle = -2f_\pi^2 M_\pi^2
\]

where \( \langle 0 | \bar{q}q | 0 \rangle = -(267 \pm 5\text{MeV})^3 \), and \( f_\pi = 92.21 \pm 0.14\text{MeV} \)
- The derivative is given by:

\[^6\text{P. Gell-Mann, R. J. Oakes, B. Renner, Phys. Rev. 175, 2195 (1968).}\]
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

$\pi$-meson

Using GMOR:\textsuperscript{6}

$$4m_q \langle 0 | \bar{q}q | 0 \rangle = -2f_\pi^2 M_\pi^2$$ \hspace{1cm} (8)

where $\langle 0 | \bar{q}q | 0 \rangle = -(267 \pm 5\text{MeV})^3$, and $f_\pi = 92.21 \pm 0.14\text{MeV}$

The derivative is given by:

$$\frac{\delta M_\pi}{\delta m_q} = \frac{\langle 0 | \bar{q}q | 0 \rangle}{M_\pi f_\pi^2}$$ \hspace{1cm} (9)

**Motivation**

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

**Results**

- **π-meson**
- **σ-meson**
- **ρ-meson and ω-meson**

### π-meson

- Using GMOR:\(^6\)

\[
4m_q \langle 0 | \bar{q}q | 0 \rangle = -2f_π^2 M_π^2
\] (8)

where \( \langle 0 | \bar{q}q | 0 \rangle = -(267 \pm 5\text{MeV})^3 \), and \( f_π = 92.21 \pm 0.14\text{MeV} \)

- The derivative is given by:

\[
\frac{\delta M_π}{\delta m_q} = \frac{\langle 0 | \bar{q}q | 0 \rangle}{M_π f_π^2}
\] (9)

- Final result:

---

π-meson

- Using GMOR:\(^6\)

\[
4m_q \langle 0 | \bar{q}q | 0 \rangle = -2f_\pi^2 M_\pi^2
\]  

(8)

where \( \langle 0 | \bar{q}q | 0 \rangle = - (267 \pm 5 \text{MeV})^3 \), and
\( f_\pi = 92.21 \pm 0.14 \text{MeV} \)

- The derivative is given by:

\[
\frac{\delta M_\pi}{\delta m_q} = \frac{\langle 0 | \bar{q}q | 0 \rangle}{M_\pi f_\pi^2}
\]

(9)

- Final result:

\[
\frac{\delta M_\pi}{\delta m_q} = 16.078 \pm 0.049.
\]

---

Using GMOR:\(^6\)

\[
4m_q \langle 0 | \bar{q} q | 0 \rangle = -2f_\pi^2 M_\pi^2
\]

(8)

where \(\langle 0 | \bar{q} q | 0 \rangle = -(267 \pm 5\text{MeV})^3\), and \(f_\pi = 92.21 \pm 0.14\text{MeV}\)

The derivative is given by:

\[
\frac{\delta M_\pi}{\delta m_q} = \frac{\langle 0 | \bar{q} q | 0 \rangle}{M_\pi f_\pi^2}
\]

(9)

Final result:

\[
\frac{\delta M_\pi}{\delta m_q} = 16.078 \pm 0.049.
\]

(10)

---

Outline

1 Motivation
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4 Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

$\sigma$-meson

- Self-energy contribution $i\Sigma_{\pi\pi}^\sigma (p^2)$:
**σ-meson**

- Self-energy contribution $i\Sigma_{\pi\pi}^\sigma (p^2)$:
**σ-meson**

- Self-energy contribution \( i\Sigma_{\pi\pi}^{\sigma}(p^2) \):

- Total propagator \( \Delta_{\sigma} \):
**σ-meson**

- Self-energy contribution $i \Sigma_{\pi\pi}^\sigma (p^2)$:

- Total propagator $\Delta_\sigma$:

\[
\begin{align*}
\pi & \quad \sigma \\
\sigma & \quad \pi \\
\end{align*}
\]
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

\(\sigma\)-meson

- **Self-energy contribution** $i\Sigma_{\pi\pi}^\sigma (p^2)$:

- **Total propagator** $\Delta^\sigma$:

\[
\begin{align*}
\pi & \quad \pi \\
\sigma & \quad \sigma \\
\pi & \quad \pi
\end{align*}
\]

\[
\begin{align*}
+ & \quad (\quad ) + \quad (\quad ) (\quad ) \\
+ & \quad (\quad ) (\quad ) (\quad ) \\
+ & \quad (\quad ) (\quad ) (\quad ) (\quad ) \ldots
\end{align*}
\]
\( \sigma \)-meson

- \( \Delta_\sigma \) converges to:
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

$\sigma$-meson

$\Delta_\sigma$ converges to:

$$\Delta_\sigma = \frac{1}{p^2 - \left( m_\sigma^{(0)} \right)^2 + \Sigma_\pi \pi (p)}$$

(11)
σ-meson

- $\Delta_\sigma$ converges to:

$$\Delta_\sigma = \frac{1}{p^2 - (m_{\sigma}^{(0)})^2 + \Sigma_{\pi\pi}^\sigma(p)}$$  \hspace{1cm} (11)

- Self-energy Calculation:
The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

Results

$\pi$-meson

$\sigma$-meson

$\rho$-meson and $\omega$-meson

$\sigma$-meson

- $\Delta_{\sigma}$ converges to:

$$\Delta_{\sigma} = \frac{1}{p^2 - \left(m_{\sigma}^{(0)}\right)^2 + \Sigma_{\pi\pi}^{\sigma}(p)}$$  \hspace{1cm} (11)$$

- Self-energy Calculation:

$$i\Sigma_{\pi\pi}^{\sigma} = \frac{3}{2} \gamma^2 \int \frac{d^4 k}{(2\pi)^4} \frac{k^\mu (p-k)_\mu}{(k^2 - m_{\pi}^2) \left( (p-k)^2 - m_{\pi}^2 \right)} \left[ 1 - \frac{(p^2 - k^2)^2}{\Lambda^2} \right]^{-4}$$  \hspace{1cm} (12)$$
**σ-meson**

- $\Delta_\sigma$ converges to:

$$\Delta_\sigma = \frac{1}{p^2 - \left( m^{(0)}_\sigma \right)^2 + \Sigma_{\pi\pi}^\sigma(p)}$$  \hspace{1cm} (11)

- Self-energy Calculation:

$$i\Sigma_{\pi\pi}^\sigma = \frac{3}{2} \gamma^2 \int \frac{d^4 k}{(2\pi)^4} \frac{\left[ k^\mu \left( p - k \right)_\mu \right]^2 \left[ 1 - \left( \frac{p - k}{\Lambda} \right)^2 \right]^{-4}}{(k^2 - m^2_\pi) \left( (p - k)^2 - m^2_\pi \right)}$$  \hspace{1cm} (12)
Motivation

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

Results

π-meson
σ-meson
ρ-meson and ω-meson

Since $\Sigma_{\pi\pi}^\sigma (\gamma, \Lambda)$ and in the centre of mass frame, the pole of the propagator must agree with the resonance.

---

Since $\Sigma_{\pi\pi}^{\sigma}(\gamma, \Lambda)$ and in the centre of mass frame, the pole of the propagator must agree with the resonance.

We fixed $\Lambda$ to assure convergence, and fit $m_{\sigma}^0$, and $\gamma$ to find the pole at the same position as Leutwyler. et al.\(^7\)


Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

- $\pi$-meson
- $\sigma$-meson
- $\rho$-meson and $\omega$-meson

Since $\Sigma_{\pi\pi}^\sigma (\gamma, \Lambda)$ and in the centre of mass frame, the pole of the propagator must agree with the resonance.

- We fixed $\Lambda$ to assure convergence, and fit $m_\sigma^0$, and $\gamma$ to find the pole at the same position as Leutwyler. et al.\textsuperscript{7}
- Why Leutwyler?

---

Since $\Sigma_{\pi\pi} (\gamma, \Lambda)$ and in the centre of mass frame, the pole of the propagator must agree with the resonance.

- We fixed $\Lambda$ to assure convergence, and fit $m_\sigma^0$, and $\gamma$ to find the pole at the same position as Leutwyler. et al.\(^7\)

- Why Leuwtyler? $\rightarrow$ Roy Equations.\(^8\)

---


Table: Parameters that were fixed in order to agree with the position of the pole. $\gamma_0 = 6.416 \times 10^{-3} \,(MeV^{-1})$.

<table>
<thead>
<tr>
<th>$\gamma ,(\times \gamma_0)$</th>
<th>$\Lambda ,(MeV)$</th>
<th>$m^{(0)}_\sigma ,(MeV)$</th>
<th>$\Delta m_\sigma ,(MeV) \times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.56</td>
<td>320.00</td>
<td>563.54</td>
<td>1.2 – 1.0i</td>
</tr>
<tr>
<td>4.60</td>
<td>330.00</td>
<td>600.23</td>
<td>1.1 – 0.5i</td>
</tr>
<tr>
<td>4.70</td>
<td>340.00</td>
<td>639.85</td>
<td>1.2 – 0.0i</td>
</tr>
<tr>
<td>4.83</td>
<td>350.00</td>
<td>683.46</td>
<td>1.2 – 0.0i</td>
</tr>
<tr>
<td>5.02</td>
<td>360.00</td>
<td>732.00</td>
<td>0.0 + 0.0i</td>
</tr>
<tr>
<td>5.27</td>
<td>370.00</td>
<td>790.18</td>
<td>0.9 – 1.2i</td>
</tr>
<tr>
<td>5.61</td>
<td>380.00</td>
<td>859.15</td>
<td>0.4 – 0.9i</td>
</tr>
<tr>
<td>6.07</td>
<td>390.00</td>
<td>945.43</td>
<td>0.9 – 0.9i</td>
</tr>
</tbody>
</table>
Table: Parameters that were fixed in order to agree with the position of the pole. $\gamma_0 = 6.416 \times 10^{-3}$ (MeV$^{-1}$).

<table>
<thead>
<tr>
<th>$\gamma \times \gamma_0$</th>
<th>$\Lambda$ (MeV)</th>
<th>$m_{\sigma}^{(0)}$ (MeV)</th>
<th>$\Delta m_{\sigma}$ (MeV) $\times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.56</td>
<td>320.00</td>
<td>563.54</td>
<td>1.2 – 1.0i</td>
</tr>
<tr>
<td>4.60</td>
<td>330.00</td>
<td>600.23</td>
<td>1.1 – 0.5i</td>
</tr>
<tr>
<td>4.70</td>
<td>340.00</td>
<td>639.85</td>
<td>1.2 – 0.0i</td>
</tr>
<tr>
<td>4.83</td>
<td>350.00</td>
<td>683.46</td>
<td>1.2 – 0.0i</td>
</tr>
<tr>
<td>5.02</td>
<td>360.00</td>
<td>732.00</td>
<td>0.0 + 0.0i</td>
</tr>
<tr>
<td>5.27</td>
<td>370.00</td>
<td>790.18</td>
<td>0.9 – 1.2i</td>
</tr>
<tr>
<td>5.61</td>
<td>380.00</td>
<td>859.15</td>
<td>0.4 – 0.9i</td>
</tr>
<tr>
<td>6.07</td>
<td>390.00</td>
<td>945.43</td>
<td>0.9 – 0.9i</td>
</tr>
</tbody>
</table>
\[ \Sigma_{\pi\pi}^\sigma \] also depended on \( M_\pi \) so we changed \( M_\pi \) near physical value and checked variations in \( \Sigma_{\pi\pi}^\sigma \): \[ \frac{\delta \Sigma_{\pi\pi}^\sigma}{\delta M_{\rho i}^2}. \]

\(^9\text{Calculated by Ian Clöet}\)
σ-meson

- $\Sigma^{\sigma}_{\pi\pi}$ also depended on $M_\pi$ so we changed $M_\pi$ near physical value and checked variations in $\Sigma^{\sigma}_{\pi\pi}$: $\frac{\delta \Sigma^{\sigma}_{\pi\pi}}{\delta M^2_\rho}$.

- $m^{(0)}_\sigma$ also depended on $m_q$.
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with \( m_q \)

Results
\( \pi \)-meson
\( \sigma \)-meson
\( \rho \)-meson and \( \omega \)-meson

\( \sigma \)-meson

- \( \Sigma^{\sigma}_{\pi\pi} \) also depended on \( M_\pi \) so we changed \( M_\pi \) near physical value and checked variations in \( \Sigma^{\sigma}_{\pi\pi} : \frac{\delta \Sigma^{\sigma}_{\pi\pi}}{\delta M^2_{\rho i}} \).
- \( m^{(0)}_{\sigma} \) also depended on \( m_q \) \( \rightarrow \) NJL model.\(^9\)

\(^9\)Calculated by Ian Clöet
Motivation

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$.

Results

- $\pi$-meson
- $\sigma$-meson
- $\rho$-meson and $\omega$-meson

$\sigma$-meson

- $\Sigma^\sigma_{\pi \pi}$ also depended on $M_\pi$ so we changed $M_\pi$ near physical value and checked variations in $\Sigma^\sigma_{\pi \pi}: \frac{\delta \Sigma^\sigma_{\pi \pi}}{\delta M^2_\rho i}$.

- $m^{(0)}_\sigma$ also depended on $m_q \rightarrow$ NJL model.\(^9\)

- Final result:

\(^9\)Calculated by Ian Clöet
σ-meson

- $\Sigma_{\pi\pi}^\sigma$ also depended on $M_\pi$ so we changed $M_\pi$ near physical value and checked variations in $\Sigma_{\pi\pi}^\sigma$: $\frac{\delta \Sigma_{\pi\pi}^\sigma}{\delta M_{\rho i}^2}$.
- $m_{\sigma}^{(0)}$ also depended on $m_q \rightarrow$ NJL model.\(^9\)
- Final result:

$$\frac{\delta m_{\sigma}(OBE)}{m_{\sigma}(OBE)} = \nu_{m_q} \frac{\delta m_q}{m_q}$$

\(9\)Calculated by Ian Clöet
**σ-meson**

- $\Sigma_{\pi\pi}^{\sigma}$ also depended on $M_\pi$, so we changed $M_\pi$ near physical value and checked variations in $\Sigma_{\pi\pi}^{\sigma}$: \[ \frac{\delta \Sigma_{\pi\pi}^{\sigma}}{\delta M_{\rho i}^2}. \]

- $m_\sigma^{(0)}$ also depended on $m_q \rightarrow$ NJL model.\(^9\)

- Final result:

\[
\frac{\delta m_\sigma(OBE)}{m_\sigma(OBE)} = \nu_{m_q} \frac{\delta m_q}{m_q}
\]

(13)

with:

\[^9\text{Calculated by Ian Clöet}\]
Motivation

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

Results

$\pi$-meson

$\sigma$-meson

$\rho$-meson and $\omega$-meson

$\sigma$-meson

- $\Sigma_{\pi\pi}^\sigma$ also depended on $M_\pi$ so we changed $M_\pi$ near physical value and checked variations in $\Sigma_{\pi\pi}^\sigma$: $\frac{\delta \Sigma_{\pi\pi}^\sigma}{\delta M_{\pi i}^2}$.

- $m^{(0)}_\sigma$ also depended on $m_q \rightarrow$ NJL model.\(^9\)

- Final result:

\[
\frac{\delta m_\sigma(OBE)}{m_\sigma(OBE)} = \nu_{m_q} \frac{\delta m_q}{m_q}
\]  

(13)

with:

\[
\nu_{m_q} = \frac{M_\pi^2}{2m_\sigma(OBE)^2} \left[ \frac{\delta \left( m^{(0)}_\sigma \right)}{\delta M_{\pi i}^2} - \frac{\delta \Sigma_{\pi\pi}^\sigma(0)}{\delta M_{\pi i}^2} \right].
\]  

(14)

---

\(^9\) Calculated by Ian Clöet
Motivation

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

Results

$\sigma$-meson

- $\Sigma_{\pi\pi}^\sigma$ also depended on $M_\pi$ so we changed $M_\pi$ near physical value and checked variations in $\Sigma_{\pi\pi}^\sigma$: $\frac{\delta \Sigma_{\pi\pi}^\sigma}{\delta M_{\pi}^2}$.

- $m_\sigma^{(0)}$ also depended on $m_q \rightarrow$ NJL model.\(^9\)

- Final result:

$$\frac{\delta m_\sigma(OBE)}{m_\sigma(OBE)} = \nu_{m_q} \frac{\delta m_q}{m_q}$$

(13)

with:

$$\nu_{m_q} = \frac{M_{\pi}^2}{2m_\sigma(OBE)^2} \left[ \frac{\delta \left( m_\sigma^{(0)} \right)}{\delta M_{\pi}^2} - \frac{\delta \Sigma_{\pi\pi}^\sigma(0)}{\delta M_{\pi}^2} \right].$$

(14)

\(^9\)Calculated by Ian Clöet
## Motivation

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

## Results

### $\pi$-meson

$\sigma$-meson

$\rho$-meson and $\omega$-meson

### $\sigma$-meson

Table: Calculations for the coefficient $\nu_{m_q}$.

<table>
<thead>
<tr>
<th>$m_{\sigma}^{(0)}$</th>
<th>$\frac{\delta \Sigma_{\pi\pi}^{\sigma} (0)}{\delta M_{\pi}^2}$</th>
<th>$\frac{\delta (m_{\sigma}^{(0)})^2}{\delta M_{\pi}^2}$</th>
<th>$\frac{\delta m_{\sigma}^2 (OBE)}{\delta M_{\pi}^2}$</th>
<th>$\nu_{m_q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>563.54</td>
<td>-0.145</td>
<td>2.677</td>
<td>2.822</td>
<td>0.089</td>
</tr>
<tr>
<td>600.23</td>
<td>-0.164</td>
<td>2.632</td>
<td>2.796</td>
<td>0.078</td>
</tr>
<tr>
<td>639.85</td>
<td>-0.189</td>
<td>2.576</td>
<td>2.765</td>
<td>0.068</td>
</tr>
<tr>
<td>683.46</td>
<td>-0.220</td>
<td>2.546</td>
<td>2.766</td>
<td>0.060</td>
</tr>
<tr>
<td>732.00</td>
<td>-0.261</td>
<td>2.502</td>
<td>2.763</td>
<td>0.052</td>
</tr>
<tr>
<td>790.18</td>
<td>-0.314</td>
<td>2.451</td>
<td>2.765</td>
<td>0.045</td>
</tr>
<tr>
<td>859.15</td>
<td>-0.389</td>
<td>2.401</td>
<td>2.790</td>
<td>0.038</td>
</tr>
<tr>
<td>945.43</td>
<td>-0.495</td>
<td>2.344</td>
<td>2.839</td>
<td>0.032</td>
</tr>
</tbody>
</table>
Table: Calculations for the coefficient $\nu_{m_q}$.

<table>
<thead>
<tr>
<th>$m_\sigma^{(0)}$</th>
<th>$\frac{\delta \Sigma_{\pi\pi}^{\sigma}(0)}{\delta M_{\pi}^2}$</th>
<th>$\frac{\delta (m_\sigma^{(0)})^2}{\delta M_{\pi}^2}$</th>
<th>$\frac{\delta m_\sigma^2(OBE)}{\delta M_{\pi}^2}$</th>
<th>$\nu_{m_q}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>563.54</td>
<td>-0.145</td>
<td>2.677</td>
<td>2.822</td>
<td>0.089</td>
</tr>
<tr>
<td>600.23</td>
<td>-0.164</td>
<td>2.632</td>
<td>2.796</td>
<td>0.078</td>
</tr>
<tr>
<td>639.85</td>
<td>-0.189</td>
<td>2.576</td>
<td>2.765</td>
<td>0.068</td>
</tr>
<tr>
<td>683.46</td>
<td>-0.220</td>
<td>2.546</td>
<td>2.766</td>
<td>0.060</td>
</tr>
<tr>
<td>732.00</td>
<td>-0.261</td>
<td>2.502</td>
<td>2.763</td>
<td>0.052</td>
</tr>
<tr>
<td>790.18</td>
<td>-0.314</td>
<td>2.451</td>
<td>2.765</td>
<td>0.045</td>
</tr>
<tr>
<td>859.15</td>
<td>-0.389</td>
<td>2.401</td>
<td>2.790</td>
<td>0.038</td>
</tr>
<tr>
<td>945.43</td>
<td>-0.495</td>
<td>2.344</td>
<td>2.839</td>
<td>0.032</td>
</tr>
</tbody>
</table>
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

Outline

1. Motivation
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2. The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3. Variations in the mass of the Mesons with $m_q$
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4. Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
We use the results from Allton, Armour, Leinweber, Thomas, and Young.\textsuperscript{10}

We use the results from Allton, Armour, Leinweber, Thomas, and Young.\textsuperscript{10}

CP-PACS collaboration results.

**Motivation**

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

**Results**

- **π-meson**
- **σ-meson**
- **ρ-meson and ω-meson**

**ρ-meson**

- We use the results from Allton, Armour, Leinweber, Thomas, and Young.\(^\text{10}\)
- CP-PACS collaboration results.

---

Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$
Results

$\rho$-meson

- We use the results from Allton, Armour, Leinweber, Thomas, and Young.\(^\text{10}\)
- CP-PACS collaboration results.

They proved that a fit of the form (Chiral expansion):
They proved that a fit of the form (Chiral expansion):

$$\sqrt{m_{\rho}^2 + \Sigma} = a_0 + a_2 M_{\pi}^2 + a_4 M_{\pi}^4 + a_6 M_{\pi}^6$$  \hspace{1cm} (15)
They proved that a fit of the form (Chiral expansion):

$$\sqrt{m^2_\rho + \Sigma} = a_0 + a_2 M^2_\pi + a_4 M^4_\pi + a_6 M^6_\pi$$  \hspace{1cm} (15)
<table>
<thead>
<tr>
<th>π-meson</th>
<th>σ-meson</th>
<th>ρ-meson and ω-meson</th>
</tr>
</thead>
</table>

**Motivation**

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

**Results**

- π-meson
- σ-meson
- ρ-meson and ω-meson

**ρ-meson**

- With $\Sigma$ representing the leading order self-energies (In partially-quenched QCD).
With $\Sigma$ representing the leading order self-energies (In partially-quenched QCD).
Every point is shifted by an amount:
Every point is shifted by an amount:

$$\delta M_\rho = \left[ \left( a_0 + a_2 M_\pi^2 + a_4 M_\pi^4 \right)^2 + \sum \left( M_\pi; L \rightarrow \infty \right) \right]^{1/2}$$

$$- \left[ \left( (a_0 + X_2 a^2) + a_2 \hat{M}_\pi^2 + a_4 \hat{M}_\pi^4 \right)^2 + \sum \left( \hat{M}_\pi; L \right) \right]^{1/2}$$
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
π-meson
σ-meson
ρ-meson and ω-meson

ρ-meson

- Improves data
**Motivation**

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

**Results**

- $\pi$-meson
- $\sigma$-meson
- $\rho$-meson and $\omega$-meson

**$\rho$-meson**

- Improves data

![Graph showing the relationship between $M_{\rho}^{\text{unit}}$ and $(M_{PS}^{\text{unit}})^2$ with different values of $\beta$.]
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

$\rho$-meson

- Improves data
Variation in physical regime gives:
Variation in physical regime gives:

\[
\frac{\delta m_\rho}{\delta m_q} = \left( \frac{M_\pi^2}{m_\rho} \frac{\delta m_\rho}{\delta M_\pi^2} \right) \frac{\delta m_q}{m_q}
\]

(16)

\[
\frac{\delta m_\rho}{\delta m_q} = 5.236.
\]

(17)
Simmilar calculation to $\rho$-meson, but with just the following diagram:
Simmilar calculation to $\rho$-meson, but with just the following diagram:

\[ \omega \quad \pi \quad \omega \quad \rho \quad \omega \quad \rho \]

\[ = 3 \times \]

\[ \pi \quad \omega \quad \rho \quad \rho \]

The other diagram does not contribute because G-parity is not conserved.
Simmilar calculation to $\rho$-meson, but with just the following diagram:

\[
\begin{align*}
\omega & \quad \pi & \quad \omega \\
\omega & \quad \rho & \quad \omega
\end{align*}
\]

\[
= 3 \times 
\begin{align*}
\pi & \quad \omega & \quad \rho \\
\rho & \quad \omega & \quad \rho
\end{align*}
\]

The other diagram does not contribute because G-parity is not conserved.
We can use the same coefficients and expansion as for the \( \rho \)-meson, because their mass difference is about 12 MeV.
We can use the same coefficients and expansion as for the \( \rho \)-meson, because their mass difference is about 12 MeV.

In the physical regime we obtain:
We can use the same coefficients and expansion as for the $\rho$-meson, because their mass difference is about 12 MeV.

In the physical regime we obtain:

$$\frac{\delta m_\omega}{\delta m_q} = 2.190.$$  (18)
Outline

1 Motivation
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2 The One Boson Exchange Potential.
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3 Variations in the mass of the Mesons with $m_q$
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4 Results
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7\text{Li}$, $^{12}\text{C}$, $^{16}\text{O}$ Nuclei
We used the Bryan-Scott potential.\textsuperscript{11}
We used the Bryan-Scott potential.\textsuperscript{11}

In terms of the exchanged mesons can be divided in:

\textsuperscript{11}Calculated by Iraj Afnan
We used the Bryan-Scott potential.\(^{11}\)

In terms of the exchanged mesons can be divided in:

\[
V_{OBE} = \sum_{\nu} V^{\nu}, (\nu = \pi, \eta, \rho, \omega, \sigma_1, \sigma_0). \quad (19)
\]

Each contribution depends on the "nature" of the meson: scalar, pseudo-scalar, vector.

\(^{11}\)Calculated by Iraj Afnan
• We used the Bryan-Scott potential.\(^\text{11}\)

• In terms of the exchanged mesons can be divided in:

\[
V_{OBE} = \sum_{\nu} V^\nu, (\nu = \pi, \eta, \rho, \omega, \sigma_1, \sigma_0).
\] (19)

• Each contribution depends on the "nature" of the meson: scalar, pseudo-scalar, vector.

\(^{11}\)Calculated by Iraj Afnan
After solving numerically the Schrödinger equation for different masses for the exchanged mesons:
After solving numerically the Schrödinger equation for different masses for the exchanged mesons:

Fit: $5.5449 - 0.0239 \, m_\pi$
After solving numerically the Schrödinger equation for different masses for the exchanged mesons:

Fit: $5.5449 - 0.0239 \, m_\pi$
Deuteron

For $m_{\sigma 0}$:
Deuteron

For $m_{\sigma 0}$:

\[
\text{Fit: } 50.0737 - 0.087 m_{\sigma 0}
\]
Deuteron

For $m_{\sigma 0}$:

$$E_D(\text{MeV}) = 50.0737 - 0.087 m_{\sigma 0}$$
Deuteron

For $m_{\sigma_1}$:
**Deuteron**

For $m_{\sigma_1}$:

Fit: $0.0465 m_{\sigma_1} - 25.6612$

\[ E_D(\text{MeV}) \]

\[ m_{\sigma_1} (\text{MeV}) \]
Deuteron

For $m_{\sigma_1}$:

Fit: $0.0465 m_{\sigma_1} - 25.6612$
Deuteron

- For $m_\rho$:
Deuteron

For $m_\rho$:

$$\text{Fit: } 24.5411 - 0.0292 \, m_\rho$$
Motivation

The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

Results

Binding energy of the Deuteron

Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei

Deuteron

For $m_\rho$:

\[ \text{Fit: } 24.5411 - 0.0292 m_\rho \]
Deuteron

- For $m_\omega$: 

Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Binding energy of the Deuteron
Single-Particle Energies for $^7\text{Li}$, $^{12}\text{C}$, $^{16}\text{O}$ Nuclei
Deuteron

- For $m_\omega$:

$$\text{Fit: } 0.0907 m_\omega - 68.7447$$
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Binding energy of the Deuteron
Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei

Deuteron

For $m_\omega$:

$$E_D(\text{MeV})$$

Fit: $0.0907 m_\omega - 68.7447$

$m_\omega$ (MeV)
Deuteron

- For $m_\eta$: 
Deuteron

For \( m_\eta \):

\[
\text{Fit} : 0.0019 \, m_\eta + 1.1686
\]
Deuteron

For $m_\eta$:

\[ \text{Fit: } 0.0019 \, m_\eta + 1.1686 \]
Deuteron

From these results we could evaluate the variation in the binding energy of the Deuteron:
From these results we could evaluate the variation in the binding energy of the Deuteron:

\[
\frac{\delta E_D}{\delta m_q} = \frac{\delta E_D}{\delta M_\pi} \frac{\delta M_\pi}{\delta m_q} + \frac{\delta E_D}{\delta m_{\sigma 0}} \frac{\delta m_{\sigma 0}}{\delta m_q} + \frac{\delta E_D}{\delta m_{\sigma 1}} \frac{\delta m_{\sigma 1}}{\delta m_q} + \frac{\delta E_D}{\delta m_\rho} \frac{\delta m_\rho}{\delta m_q} + \frac{\delta E_D}{\delta m_\omega} \frac{\delta m_\omega}{\delta m_q}
\]

The final result is:

\[
47
\]
From these results we could evaluate the variation in the binding energy of the Deuteron:

\[
\frac{\delta E_D}{\delta m_q} = \frac{\delta E_D}{\delta M_\pi} \frac{\delta M_\pi}{\delta m_q} + \frac{\delta E_D}{\delta m_{\sigma 0}} \frac{\delta m_{\sigma 0}}{\delta m_q} + \frac{\delta E_D}{\delta m_{\sigma 1}} \frac{\delta m_{\sigma 1}}{\delta m_q} + \frac{\delta E_D}{\delta m_\rho} \frac{\delta m_\rho}{\delta m_q} + \frac{\delta E_D}{\delta m_\omega} \frac{\delta m_\omega}{\delta m_q}
\]

(20)

The final result is:

\[
\frac{\delta E_D}{\delta m_q} = -1.1604
\]

(21)
**Outline**

1. **Motivation**
   - Experimental Evidence
   - Time shifts in $\alpha$ and its implications on the nucleon mass

2. **The One Boson Exchange Potential.**
   - Range of the potential
   - Schematic Representation
   - Implications on Binding Energies

3. **Variations in the mass of the Mesons with $m_q$**
   - $\pi$-meson
   - $\sigma$-meson
   - $\rho$-meson and $\omega$-meson

4. **Results**
   - Binding energy of the Deuteron
   - Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei
Calculation using the Quark-Meson-Coupling (QMC) model.\textsuperscript{12}
Calculation using the Quark-Meson-Coupling (QMC) model.\textsuperscript{12}

Since the QMC model couples the exchanged mesons to the quarks that composed the nucleon, if we change the masses of those mesons we obtain different values in the calculation for binding energies.

\textsuperscript{12}Calculated by Kazuo Tsushima
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Binding energy of the Deuteron
Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei

\[
\begin{array}{ccccccc}
4.90 & 4.95 & 5.00 & 5.05 & 5.10 \\
\end{array}
\]

\[
\begin{array}{ccccccc}
5.00 & 5.05 & 5.10 & 5.15 & 5.20 \\
\end{array}
\]
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

Binding energy of the Deuteron

Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei

$E(\text{MeV})$/Nucleon

$^7$Li with $\frac{\delta(E/A)}{\delta m_q} = 1.04$
Motivation
The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results
Binding energy of the Deuteron
Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei

$$E(\text{MeV})/\text{Nucleon}$$

$^7$Li with $\frac{\delta(E/A)}{\delta m_q} = 1.04$ $^{12}$C with $\frac{\delta(E/A)}{\delta m_q} = 1.45$
The One Boson Exchange Potential.

Variations in the mass of the Mesons with $m_q$

Results

Binding energy of the Deuteron

Single-Particle Energies for $^7\text{Li}$, $^{12}\text{C}$, $^{16}\text{O}$ Nuclei

\[
\begin{align*}
\frac{\delta(E/A)}{\delta m_q} & = 1.04 & \quad ^7\text{Li} \\
\frac{\delta(E/A)}{\delta m_q} & = 1.45 & \quad ^{12}\text{C} \\
\frac{\delta(E/A)}{\delta m_q} & = 1.61 & \quad ^{16}\text{O}
\end{align*}
\]
Motivation

The One Boson Exchange Potential.
Variations in the mass of the Mesons with $m_q$

Results

Binding energy of the Deuteron

Single-Particle Energies for $^7$Li, $^{12}$C, $^{16}$O Nuclei

$E(\text{MeV})$ per Nucleon

$m_q(\text{MeV})$

$^7$Li with $\frac{\delta(E/A)}{\delta m_q} = 1.04$
$^{12}$C with $\frac{\delta(E/A)}{\delta m_q} = 1.45$
$^{16}$O with $\frac{\delta(E/A)}{\delta m_q} = 1.61$
Conclusions

- Big Bang Nucleosynthesis is a successful prediction of the big-bang model.
- It predicts the abundance of light elements $^1D$ and $^4He$ with good agreement with experimental observations.
- However for $^7Li$ there is a discrepancy. It is predicted to be:

$$^7Li/H = 4.15^{+0.49}_{-0.45} \times 10^{-10}$$

(22)

but observations give values of:

$$^7Li/H = (1.26 \pm 0.26) \times 10^{-10}$$

(23)

- Since BBN depends on the binding energy, if it is changing in cosmological time that could be an answer to the difference.\(^\text{13}\)

Conclusions

- Big Bang Nucleosynthesis is a successful prediction of the big-bang model.
- It predicts the abundance of light elements $D$ and $^4He$ with good agreement with experimental observations.
- However for $^7Li$ there is a discrepancy. It is predicted to be:

$$^7Li/H = 4.15^{+0.49}_{-0.45} \times 10^{-10} \quad (22)$$

but observations give values of:

$$^7Li/H = (1.26 \pm 0.26) \times 10^{-10} \quad (23)$$

- Since BBN depends on the binding energy, if it is changing in cosmological time that could be an answer to the difference.$^{13}$

Variations in the fine structure constant can be linked to changes in other fundamental quantities through GUTS.

If $m_q$ changes then the mass of the exchanged mesons in the OBE potential of the nucleon-nucleon interaction change.

Therefore binding energies are affected.

This could give rise to modifications of BBN.
Thanks