The Quark-Meson Coupling model as a description of dense matter

J. D. Carroll

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Outline

1. The Basics:
   - The Nature of Dense Matter
   - The Models

2. Simulations:
   - Hadronic Matter
   - Mixed-Phase Matter
We wish to understand the properties of matter over a wide range of densities; from single atomic nuclei to neutron stars.

In order to do this, we must first understand the fundamental and effective degrees of freedom at each density scale.

In order to evaluate success, we need to compare predictions over a wide range of density scales to observable phenomena.
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What we know:

- Quarks and Gluons are the fundamental degrees of freedom.
- At low densities, Baryons (Nucleons) are the effective degrees of freedom.
- At high densities...
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QMC dense matter
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- At low densities, Baryons (Nucleons) are the effective degrees of freedom
- At high densities... ??? = Hyperons? Quarks? Other?
Quantum HadroDynamics (QHD) Model

- Simple description of nucleons immersed in mean-field $\sigma$, $\omega$, and $\rho$ potentials,
- Constructed at the baryon level,
- Issues with large scalar potentials causing negative effective masses.

Quark-Meson Coupling (QMC) Model

- Similar final form as QHD, but with self-consistent response to the $\sigma$ field, despite construction from quark level,
- Better predictions for bulk properties of dense matter,
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Hadronic Models
A Brief Overview

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M_B^* = M_B + \Sigma_B = M_B - g_{\sigma B} \langle \sigma \rangle
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M_B^* = M_B + \Sigma_B = M_B - w_B^s g_{\sigma N} \langle \sigma \rangle + \frac{d}{2} \tilde{w}_B^s (g_{\sigma N} \langle \sigma \rangle)^2
\]
In QHD, at Hartree level (mean-field), the scalar self-energy involves only the tadpole term:
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\[ \Sigma^s_B (\text{QHD}) \]

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\Sigma^s_B = -g_{\sigma B} \langle \sigma \rangle \\
= -g_{\sigma B} \sum_{B'} \frac{g_{\sigma B'}}{m^2_{\sigma}} \frac{(2J_{B'} + 1)}{(2\pi)^3} M^*_B \int \frac{\theta(k_{F_{B'}} - |\vec{k}|)}{\sqrt{\vec{k}^2 + M^*_B}} d^3 k
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Hyperonic QMC

- \( B \in \{p, n, \Lambda, \Sigma^-, \Sigma^0, \Sigma^+, \Xi^-, \Xi^0\} = \{N, Y\} \)
- \( \ell \in \{e^-, \mu^-\} \)
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Saturation

\[
\begin{align*}
(E/A)_{\rho_0} &= -15.86 \text{ MeV,} \\
(\rho_{\text{total}})_{\rho_0} &= 0.16 \text{ fm}^{-3} \\
(a_{\text{sym}})_{\rho_0} &= 32.5 \text{ MeV}
\end{align*}
\]

\[\begin{array}{ll}
\text{---} & g_{\sigma N}, g_{\omega N} \\
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"\(g_{\sigma N}, g_{\omega N}\)" and "\(g_{\rho N}\)"

Effective masses from Ref. [4]: Guichon et. al. doi:10.1016/j.nuclphysa.2008.10.001 (previously from Ref. [5]: Rikovska-Stone et. al. doi:10.1016/j.nuclphysa.2007.05.011) derived from the bag model.
Equation of State (EOS) is calculated assuming that

\[ \mu_i = B_i \mu_n - Q_i \mu_e = \sqrt{\frac{k^2}{F_i} + (M_i + \Sigma_i^s)^2 + \Sigma_i^0} \]
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Hadronic Models
A Brief Overview: Finite Nuclei

Finite Nuclei:
The mean-fields $\langle m \rangle$ are calculated via the equations of motion:

\[
\Box + m^2_\sigma \sigma = g_N \sigma \bar{\psi} \psi,
\]

\[
\partial^\mu \Omega_{\mu\nu} = g_N \omega \bar{\psi} \gamma_\nu \psi - m^2_\omega \omega_\nu,
\]

\[
\partial^\mu R^a_{\mu\nu} = g_\rho \bar{\psi} \gamma_\nu \tau^a \psi - m^2_\rho \rho^a_\nu.
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**Equations of Motion**

\[
(-\nabla^2 + m^2_\sigma) \sigma(x) = -g_N \sigma \text{Tr}[iG_H(x, x)],
\]

\[
(-\nabla^2 + m^2_\omega) \omega^\mu(x) = -g_N \omega \text{Tr}[i\gamma^\mu G_H(x, x)],
\]

\[
(-\nabla^2 + m^2_\rho) \rho^{\mu a}(x) = -g_\rho \text{Tr}[i\tau^a \gamma^\mu G_H(x, x)].
\]
Consider the solutions of the Dirac equation to be written as

\[ U_{\alpha}(x) = U_{\kappa m t}(x) = \begin{bmatrix} \frac{iG_{\kappa t}(r)}{r} & \Phi_{\kappa m t} \\ -\frac{F_{\kappa t}(r)}{r} & \Phi_{-\kappa m t} \end{bmatrix} \]
### Equations of Motion

\[
\frac{d^2}{dr^2} \sigma_0(r) + \frac{2}{r} \frac{d}{dr} \sigma_0(r) - m_\sigma^2 \sigma_0(r) = -g_N \sigma \sum_{\alpha}^{\text{occ}} \left( \frac{2j_\alpha + 1}{4\pi r^2} \right) \left( |G_\alpha(r)|^2 - |F_\alpha(r)|^2 \right),
\]

\[
\frac{d}{dr} G_\alpha(r) + \frac{\kappa}{r} G_\alpha(r) - \left[ \epsilon_\alpha - g_N \omega_0(r) - \tau_\alpha g_\rho \rho_0(r) + M^*(r) \right] = 0
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\int_0^\infty dr \left( |G_\alpha(r)|^2 + |F_\alpha(r)|^2 \right) = 1
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Hadronic Models
A Brief Overview: Finite Nuclei

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We obtain:

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- $\Rightarrow \rho_p(r), \rho_n(r), \rho_B(r)$
- $\epsilon_\alpha$, masses/splittings
- hypernuclei data

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Hyperonic QMC (2007)
Hyperonic QMC (2008)

Species Density Fraction ($Y$)

Density $\rho$ [fm$^{-3}$]

- $p^+$
- $n$
- $e^-$
- $\mu^-$
- $\Lambda$
- $\Sigma^+$
- $\Sigma^-$
- $\Sigma$
- $\Xi^-$
- $\Xi^0$
- $u$
- $d$
- $s$
The improvement in the 2008 parameterization of $M^*$ is that the effect of the mean scalar field in-medium on the familiar one-gluon-exchange hyperfine interaction (that in free space leads to the $N$-$\Delta$ and $\Sigma$-$\Lambda$ mass splittings) is also included self-consistently.

This has the effect of increasing the splitting between the $\Lambda$ and $\Sigma$ masses as the density rises and the prime reason why we find that the $\Sigma$ hypernuclei are unbound.

Guichon, Thomas, Tsushima: 2008
QMC - Finite Nuclei

1\textsuperscript{st} 1/2– Neutron Level in $^{20}\text{Ne}$

2\textsuperscript{nd} 1/2– Neutron Level in $^{20}\text{Ne}$

QMC - Finite Nuclei

QMC - Finite Nuclei

# QMC - Finite Nuclei

<table>
<thead>
<tr>
<th></th>
<th>$E_B$ (MeV) [experiment]</th>
<th>$E_B$ (MeV) [QMC]</th>
<th>$r_c$ (fm) [experiment]</th>
<th>$r_c$ (fm) [QMC]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{16}\text{O}$</td>
<td>7.976</td>
<td>7.618</td>
<td>2.73</td>
<td>2.702</td>
</tr>
<tr>
<td>$^{40}\text{Ca}$</td>
<td>8.551</td>
<td>8.213</td>
<td>3.485</td>
<td>3.415</td>
</tr>
<tr>
<td>$^{48}\text{Ca}$</td>
<td>8.666</td>
<td>8.343</td>
<td>3.484</td>
<td>3.468</td>
</tr>
<tr>
<td>$^{208}\text{Pb}$</td>
<td>7.867</td>
<td>7.515</td>
<td>5.5</td>
<td>5.42</td>
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Equations describe a static, spherically symmetric, non-rotating star, stable against gravitational collapse;

\[
\frac{dP}{dr} = -\frac{G \left( P/c^2 + \mathcal{E} \right) \left( M(r) + 4r^3\pi P/c^2 \right)}{r(r - 2GM(r)/c^2)}
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M(R) = \int_0^R 4\pi r^2 \mathcal{E}(r) \, dr
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Tolman-Oppenheimer-Volkoff

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\]
Equations describe a static, spherically symmetric, non-rotating star, stable against gravitational collapse;

\[
\frac{dP}{dr} = -\frac{G \left( \frac{P}{c^2} + \mathcal{E} \right) \left( M(r) + 4r^3\pi P/c^2 \right)}{r \left( r - 2GM(r)/c^2 \right)}
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*The data you are looking for...
Hyperonic QMC
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What about higher densities?

“What about higher densities?”
Quark Models
A Brief Overview

MIT Bag Model

- 3 quarks in a ‘bag’,
- Separated from the QCD vacuum by an energy-density $B$,
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- Simple inclusion of $D\chi$SB,
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**A Brief Overview**

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**NJL:**

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\begin{align*}
    m_q^* &= m_q + \Sigma_q = m_q - 2G\langle \bar{\psi}_q \psi_q \rangle \\
    &= m_q + \frac{8G N_c}{(2\pi)^3} \int \frac{\theta(k_F - |\vec{k}|)\theta(\Lambda - k_F)m_q^*}{\sqrt{\vec{k}^2 + m_q^*}} 
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\[ k_F = 0 \quad : \quad m_u = 350 \text{ MeV}, \quad m_d = 350 \text{ MeV}, \quad m_s = 450 \text{ MeV} \]
\[ k_F = \Lambda \quad : \quad m_u = 5 \text{ MeV}, \quad m_d = 7 \text{ MeV}, \quad m_s = 95 \text{ MeV} \]
Quark Models
NJL Effective Masses

![Graph showing dynamic quark mass vs. \( k_F \) in GeV]

- \( M_{u,d} \)
- \( M_s \)
Phase Transitions

The Gibbs Conditions for a phase transition are:

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$T = 0$
Hyperonic QMC
Phase Transition

![Graph showing species fraction $Y_i$ vs. density $\rho$ in fm$^{-3}$]

- n (proton)
- p$^+$ (proton)
- d (deuteron)
- p (proton)
- s (strange quark)
- u (up quark)
- e$^-$ (electron)
- $\mu^-$ (muon)
- $\Lambda$ (Lambda baryon)
Hyperonic QMC
Phase Transition

![Graph showing species fraction vs. density](image-url)
Hyperonic QMC
Phase Transition

\[ \rho_i \text{ [fm}^{-3}\text{]} \]

- Quark Phase
- Hadron Phase
- Total

\[ \chi \]

J. D. Carroll
QMC dense matter
Quark Chemical Potentials related to independent chemical potentials;

\[ \mu_i = B_i \mu_n - Q_i \mu_e = \sqrt{k_{F_i}^2 + M_i^2} \]

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J. D. Carroll

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Quark Phase
Hadron Phase
Total
**Nucleonic QMC**

Phase Transition

```
Species Fraction $Y_i$

Density $\rho$ [fm$^{-3}$]
```

- $n$ (neutron)
- $p^+$ (proton)
- $e^-$ (electron)
- $d$ (deuteron)
- $s$ (strange)
- $\mu^-$ (muon)
- $u$ (up)

J. D. Carroll  QMC dense matter
Mixed-Phase Hyperonic QMC

TOV solutions
The inclusion of $D\chi$SB prevents a phase transition to quark matter,

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\[
\Sigma^s_B(k) = -g_{\sigma B} \langle \sigma \rangle \\
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\times \left[ \frac{1}{4} g_{\sigma B'}^2 \Theta_\sigma(k, q) - g_{\omega B'}^2 \Theta_\omega(k, q) \right] \, dq
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Hadronic Models
A Brief Overview: Hartree–Fock

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Hadronic Models
A Brief Overview: Hartree–Fock

OR. . . $m = \langle m \rangle + \delta m$
OR... \( m = \langle m \rangle + \delta m \)

Hartree–Fock \( H_s \) (QHD)

\[
H_\sigma = \int d\vec{r} \left[ E(\langle \sigma \rangle) - \frac{1}{2} \langle \sigma \rangle \langle \frac{\partial E}{\partial \langle \sigma \rangle} \rangle + \frac{1}{2} \delta \sigma \left( \frac{\partial E}{\partial \langle \sigma \rangle} - \langle \frac{\partial E}{\partial \langle \sigma \rangle} \rangle \right) \right]
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Further Reading I

Carroll

*Applications of the Octet Baryon Quark-Meson Coupling Model to Hybrid Stars (PhD Thesis).*

Carroll, Thomas

in preparation

Carroll, Leinweber, Williams, Thomas

*Phase Transition from QMC Hyperonic Matter to Deconfined Quark Matter.*
*Phys.Rev.C79:045810, 2009*
Further Reading II

Guichon, Thomas, Tsushima
*Binding of hypernuclei in the latest quark-meson coupling model.*

Rikovska-Stone, Guichon, Matevosyan, Thomas
*Cold uniform matter and neutron stars in the quark-mesons-coupling model.*
[doi:10.1016/j.nuclphysa.2007.05.011]
Thank You!

$\text{Fin}$