A covariant description of nucleon [and nuclear] spin structure and TMDs

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**DIS on Nuclear Targets**

- Why nuclear targets?
  - only targets with \( J > \frac{1}{2} \) are nuclei
  - study QCD and nucleon structure at finite density

- Hadronic Tensor: in Bjorken limit & Callen-Gross \((F_2 = 2x F_1)\)
  - For \( J = \frac{1}{2} \) target
    \[
    W_{\mu\nu} = \left( g_{\mu\nu} \frac{p\cdot q}{q^2} + \frac{p_\mu p_\nu}{p\cdot q} \right) F_2(x, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p\cdot q} g_1(x, Q^2)
    \]
  - For arbitrary \( J \): \(- J \leq H \leq J\) \([2J + 1 \text{ EM structure functions}]\)
    \[
    W^H_{\mu\nu} = \left( g_{\mu\nu} \frac{p\cdot q}{q^2} + \frac{p_\mu p_\nu}{p\cdot q} \right) F_{2A}^H(x_A, Q^2) + \frac{i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda p^\sigma}{p\cdot q} g_{1A}^H(x_A, Q^2)
    \]

- Parton model expressions \([2J + 1 \text{ quark distributions}]\)
  \[
  g_{1A}^H(x_A) = \frac{1}{2} \sum_q e_q^2 \left[ \Delta q_A^H(x_A) + \Delta \bar{q}_A^H(x_A) \right]; \quad \text{parity} \quad \Rightarrow \quad g_{1A}^H = -g_{1A}^{-H}
  \]
Finite nuclei quark distributions

Definition of finite nuclei quark distributions

$$\Delta q^H_A(x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{iP^+ x_A \xi^- / A} \langle A, P, H | \psi_q(0) \gamma^+ \gamma_5 \psi_q(\xi^-) | A, P, H \rangle$$

Approximate using a modified convolution formalism

$$\Delta q^H_A(x_A) = \sum_{\alpha,\kappa,m} \int dy_A \int dx \delta(x_A - y_A x) \Delta f^{(H)}_{\alpha,\kappa,m}(y_A) \Delta q_{\alpha,\kappa}(x)$$
Finite nuclei quark distributions

- Definition of finite nuclei quark distributions

\[
\Delta q_A^H (x_A) = \frac{P^+}{A} \int \frac{d\xi^-}{2\pi} e^{i(P^+ x_A \xi^- / A}\langle A, P, H | \bar{\psi}_q(0) \gamma^+ \gamma_5 \psi_q(\xi^-) | A, P, H \rangle
\]

- Approximate using a modified convolution formalism

\[
\Delta q_A^H (x_A) = \sum_{\alpha, \kappa, m} \int dy_A \int dx \, \delta(x_A - y_A x) \Delta f^{(H)}_{\alpha, \kappa, m}(y_A) \Delta q_{\alpha, \kappa}(x)
\]

- Convolution formalism diagrammatically:
Convolution Formalism: implications

- Assume all spin is carried by the valence nucleons
  - If $A \gtrsim 8$ and for example if: $J = \frac{3}{2}$ \implies $F_{2A}^{3/2} \simeq F_{2A}^{1/2}$

- Basically a model independent result within the convolution formalism

- Introduce multipole quark distributions

\[
\Delta q^{(K)}(x) \equiv \sum_H (-1)^{J-H} \sqrt{2K + 1} \left( \begin{array}{c} J \\ H \end{array} \right) \left( \begin{array}{c} J \\ -H \end{array} \right) \Delta q^H(x), \quad K = 1, 3, \ldots, 2J
\]

- $J = \frac{3}{2}$ \implies $\Delta q^{(0)} = \frac{1}{\sqrt{5}} \left[ 3 \Delta q^{3/2} + \Delta q^{1/2} \right]$ \quad $\Delta q^{(2)} = \frac{1}{\sqrt{5}} \left[ \Delta q^{3/2} - 3 \Delta q^{1/2} \right]$

- Higher multipoles encapsulate difference between helicity distributions
Some multipole quark distributions results

Large $K > 1$ multipole PDFs would be very surprising

large off-shell effects &/or non-nucleon components, etc
New Sum Rules

- Sum rules for multipole quark distributions

\[
\int dx \, x^{n-1} \, q^{(K)}(x) = 0, \quad K, n \text{ even}, \quad 2 \leq n < K, \\
\int dx \, x^{n-1} \, \Delta q^{(K)}(x) = 0, \quad K, n \text{ odd}, \quad 1 \leq n < K.
\]

- Examples:

\[
J = \frac{3}{2} \implies \left\langle \Delta q^{(3)}(x) \right\rangle = 0
\]

\[
J = 2 \implies \left\langle \Delta q^{(3)}(x) \right\rangle = \left\langle q^{(4)}(x) \right\rangle = 0
\]

\[
J = \frac{5}{2} \implies \left\langle \Delta q^{(3)}(x) \right\rangle = \left\langle q^{(4)}(x) \right\rangle = \left\langle \Delta q^{(5)}(x) \right\rangle = \left\langle x^2 \, \Delta q^{(5)}(x) \right\rangle = 0
\]

- Sum rules place tight constraints on multipole PDFs

Nambu–Jona-Lasinio Model

- Interpreted as low energy chiral effective theory of QCD

\[ \frac{Z(k^2)}{k^2} \]

- Can be motivated by infrared enhancement of quark–gluon interaction
e.g. DSEs and Lattice QCD

- Investigate the role of quark degrees of freedom

- NJL has same flavour symmetries as QCD

- Lagrangian:

\[ \mathcal{L}_{NJL} = \overline{\psi}_q (i\gamma^\mu \partial_\mu - m_q) \psi_q + G (\overline{\psi}_q \Gamma \psi_q)^2 \]
Gap Equation & Mass Generation

- Quark propagator:
  \[
  \frac{1}{\not p - m + i\varepsilon} \rightarrow \frac{1}{\not p - M + i\varepsilon}
  \]

- Mass is generated via interaction with vacuum

- Dynamically generated quark masses \( \iff \langle \overline{\psi}\psi \rangle \neq 0 \)

- Proper-time regularization: \( \Lambda_{IR} \) and \( \Lambda_{UV} \)

\[ Z(p^2 = M^2) = 0 \implies \text{No free quarks} \implies \text{Confinement} \]
Nucleon in the NJL model

- Nucleon approximated as quark-diquark bound state
- Use relativistic Faddeev approach:

\[
P_k = P_{p-k} = P_{p-k}
\]

- Nucleon quark distributions

\[
q(x) = p^+ \int \frac{d\xi^-}{2\pi} e^{ixp^+\xi^-} \langle p, s|\bar{\psi}_q(0)\gamma^+\psi_q(\xi^-)|p, s\rangle_c, \quad \Delta q(x) = \langle \gamma^+\gamma_5 \rangle
\]

- Associated with a Feynman diagram calculation

\[
\sum [q(x), \Delta q(x), \Delta T q(x)] \rightarrow X = \delta \left( x - \frac{k^+}{p^+} \right) [\gamma^+, \gamma^+\gamma_5, \gamma^+\gamma^1\gamma_5]
\]
Results: proton quark distributions

Covariant, correct support, satisfies baryon and momentum sum rules

\[ \int dx \left[ q(x) - \bar{q}(x) \right] = N_q, \quad \int dx \, x \left[ u(x) + d(x) + \ldots \right] = 1 \]

Satisfies positivity constraints and Soffer bound

\[ |\Delta q(x)|, \, |\Delta_T q(x)| \leq q(x), \quad q(x) + \Delta q(x) \geq 2 |\Delta_T q(x)| \]

Why is Transversity Interesting?

\[ \Delta_T q(x) = + - \]  

- Quarks in eigenstates of \( \gamma^\perp \gamma_5 \)

- Tensor charge [c.f. Bjorken sum rule for \( g_A \)]

\[ g_T = \int dx [\Delta_T u(x) - \Delta_T d(x)] \]

- In non-relativistic limit: \( \Delta_T q(x) = \Delta q(x) \)
  - therefore \( \Delta_T q(x) \) is a measure of relativistic effects

- Helicity conservation \( \implies \) no mixing between \( \Delta_T q \) & \( \Delta_T g \)

- For \( J \leq \frac{1}{2} \) we have \( \Delta_T g(x) = 0 \)

- Therefore for the nucleon \( \Delta_T q(x) \) is valence quark dominated

- Transversity moment \( \neq \) spin quarks in transverse direction [c.f. \( g_T(x) \)]
\( \Delta_T u_v(x) \) and \( \Delta_T d_v(x) \) distributions

![Graph showing transversity distributions](image)

- Predict small relativistic corrections
- Empirical analysis *potentially* found large relativistic corrections
- Large effects difficult to support with quark mass \( \sim 0.4 \text{ GeV} \)
  - maybe running quark mass is needed
Transversity: Reanalysis

\[ Q^2 = 2.4 \text{ GeV}^2 \]

\[ x \Delta_T q_v(x) \]

\[ x \Delta_q(x) \]

\[ x \Delta_T u_v(x) \]

\[ x \Delta_T d_v(x) \]

- Our results are now in better agreement with updated distributions
- Concept of constituent quark models safe . . . for now
Transversity Moments

Our Result

- At model scale we find tensor charge

\[ g_T = 1.28 \quad \text{compared with} \quad g_A = 1.267 \]
Including Anti-quarks

- Dress quarks with pions

\[ Z_q \times \begin{array}{c} \beta \\ \alpha \end{array} p \begin{array}{c} \beta \\ \alpha \end{array} p + \begin{array}{c} \beta \\ \alpha \end{array} p \begin{array}{c} \beta \\ \alpha \end{array} p - k + \begin{array}{c} \beta \\ \alpha \end{array} p \begin{array}{c} \beta \\ \alpha \end{array} p \begin{array}{c} \beta \\ \alpha \end{array} p - k \begin{array}{c} \beta \\ \alpha \end{array} p \end{array} \]

- Constituent up quark PDFs

- Constituent down quark PDFs

- Gottfried Sum Rule: NMC 1994: \( S_G = 0.258 \pm 0.017 \) \([Q^2 = 4 \text{GeV}^2]\)

\[
S_G = \int_0^1 \frac{dx}{x} \left[ F_{2p}(x) - F_{2n}(x) \right] = \frac{1}{3} - \frac{2}{3} \int_0^1 dx \left[ \bar{d}(x) - \bar{u}(x) \right]
\]

- We find: \( S_G = \frac{1}{3} - \frac{4}{9} (1 - Z_q) = 0.252 \) \([Z_q = 0.817]\)
**Spin Sum in NJL Model**

- Nucleon angular momentum must satisfy: \[ J = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + J_g \]

\[ \Delta \Sigma = 0.33 \pm 0.03 \text{(stat.)} \pm 0.05 \text{(syst.)} \]

[COMPASS & HERMES]

- Result from Faddeev calculation: \[ \Delta \Sigma = 0.66 \]

- Correction from pion cloud: \[ \Delta \Sigma = 0.79 \times 0.66 = 0.52 \]

- Bare operator \( \gamma^\mu \gamma^5 \) gets renormalized: \[ \Delta \Sigma = 0.91 \times 0.52 = 0.47 \]
\[ \Delta f_{sq}(x) = \overline{\Gamma}_N(p) \int \frac{d^4 k}{(2\pi)^4} \delta \left( x - \frac{k^+}{p^+} \right) S(k) \gamma^+ \gamma_5 S(k) \tau_s(p - k) \Gamma_N(p) \]

- For TMDs simply do not integrate over \( \vec{k}_\perp \) – have so far \( q(x, \vec{k}_\perp^2) \)
\( p_T \) dependence

\[
\langle p_T \rangle (x) = \frac{\int d\bar{k}_\perp k_\perp q(x,k^2_\perp)}{\int d\bar{k}_\perp q(x,k^2_\perp)}
\]

For the average \( p_T \) we find

\[
\langle p_T \rangle_u = 0.36 \text{ GeV} \quad \langle p_T \rangle_d = 0.37 \text{ GeV}
\]

This compares with values derived from data

\[
\langle p_T \rangle_{\text{Gauss}} (x) = 0.64 \text{ GeV} \ [\text{EMC}] \quad \langle p_T \rangle_{\text{Gauss}} (x) = 0.56 \text{ GeV} \ [\text{HERMES}]
\]

The $N^*$ (Roper) Resonance

- $N^*$ manifests as second pole in Faddeev equation kernel
  - $M_N = 0.940 \text{ GeV}$ and $M_{N^*} = 1.8 \text{ GeV}$
  - Agrees very well with EBAC value for quark core mass
- Vertex function is given by eigenvector at pole: $p^2 = m_i^2$
- For our NJL model $N$, $N^*$ vertex function has the simple form

$$\Gamma(p) = \left[ \frac{\alpha_1}{\alpha_2 \frac{p\mu}{M} \gamma_5 + \alpha_3 \gamma^\mu \gamma_5} \right] u(p)$$

- For the nucleon: $\alpha_1 = 0.43, \alpha_2 = 0.024, \alpha_3 = -0.45$
- For the Roper: $\alpha_1 = 0.0011, \alpha_2 = 0.94, \alpha_3 = -0.051$
- $N^*$ is completely dominated by the axial–vector diquark
- $\Delta\Sigma_N = 0.68 - 0.21 = 0.47, \quad \Delta\Sigma_{N^*} = -0.02 + 0.01 \approx 0.0$
Nuclear Matter

- Finite density Lagrangian: add $\bar{q}q$ interaction in $\sigma$, $\omega$, $\rho$ channels

$$\mathcal{L} = \overline{\psi}q \left( i \slashed{\partial} - M^* - V_q \right) \psi_q + \mathcal{L}'_I$$

- Fundamental physics: mean fields couple to the quarks in nucleons

- Finite density quark propagator

$$S(k)^{-1} = k - M - i\varepsilon \quad \Rightarrow \quad S_q(k)^{-1} = k - M^* - V_q - i\varepsilon$$

- Hadronization + mean-field $\implies$ effective potential that provides

$$V_{u(d)} = \omega_0 \pm \rho_0, \quad \omega_0 = 6G_\omega (\rho_p + \rho_n), \quad \rho_0 = 2G_\rho (\rho_p - \rho_n)$$

- $G_\omega \Leftrightarrow Z = N$ saturation & $G_\rho \Leftrightarrow$ symmetry energy
Finite nuclei EMC effects

- **EMC ratio**

\[ R = \frac{F_{2A}}{F_{2A}^{\text{naive}}} = \frac{F_{2A}}{Z F_{2p} + N F_{2n}} \]

- **Polarized EMC ratio**

\[ R_s^H = \frac{g_{1A}^H}{g_{1A}^{H,\text{naive}}} = \frac{g_{1A}^H}{P_p^H g_{1p} + P_n^H g_{1n}} \]

- **Spin-dependent cross-section is suppressed by \(1/A\)**
  - Must choose nuclei with \(A \lesssim 27\)
  - Protons should carry most of the spin e.g. \(\rightarrow ^7\text{Li}, ^{11}\text{B}, \ldots\)

- **Ideal nucleus is probably \(^7\text{Li}\)**
  - From Quantum Monte–Carlo: \(P_p^J = 0.86\) & \(P_n^J = 0.04\)

- **Ratios equal 1 in non-relativistic and no-medium modification limit**
EMC Ratios

$Q^2 = 5 \text{ GeV}^2$

$Q^2 = 10.0 \text{ GeV}^2$

$\rho = 0.16 \text{ fm}^{-3}$

Is there medium modification

![Graph showing EMC Ratios for 27Al]

- Experiment: $^{27}\text{Al}$
- Unpolarized EMC effect
- Polarized EMC effect

$Q^2 = 5 \text{ GeV}^2$
Is there medium modification

- Medium modification of nucleon has been switched off
- Relativistic effects remain
- Large splitting would be strong evidence for medium modification
Nuclear Spin Sum

<table>
<thead>
<tr>
<th>Proton spin states</th>
<th>$\Delta u$</th>
<th>$\Delta d$</th>
<th>$\Sigma$</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p$</td>
<td>0.97</td>
<td>-0.30</td>
<td>0.67</td>
<td>1.267</td>
</tr>
<tr>
<td>$^7\text{Li}$</td>
<td>0.91</td>
<td>-0.29</td>
<td>0.62</td>
<td>1.19</td>
</tr>
<tr>
<td>$^{11}\text{B}$</td>
<td>0.88</td>
<td>-0.28</td>
<td>0.60</td>
<td>1.16</td>
</tr>
<tr>
<td>$^{15}\text{N}$</td>
<td>0.87</td>
<td>-0.28</td>
<td>0.59</td>
<td>1.15</td>
</tr>
<tr>
<td>$^{27}\text{Al}$</td>
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</tr>
<tr>
<td>Nuclear Matter</td>
<td>0.79</td>
<td>-0.26</td>
<td>0.53</td>
<td>1.05</td>
</tr>
</tbody>
</table>

Angular momentum of nucleon: $J = \frac{1}{2} = \frac{1}{2} \Delta \Sigma + L_q + J_g$

- in medium $M^* < M$ and therefore quarks are more relativistic
- lower components of quark wavefunctions are enhanced
- quark lower components usually have larger angular momentum
- $\Delta q(x)$ very sensitive to lower components

Conclusion: quark spin $\rightarrow$ orbital angular momentum in-medium
Conclusion

- Illustrated the inclusion of quarks into a traditional description of nuclei
  - complementary approach to traditional nuclear physics

- NJL model is a useful tool to study nucleon and nuclear structure
  - covariant, confining, dynamical chiral symmetry breaking

- NJL gives a good description of Twist-2 PDFs
  - soon have results for twist 3 and 4 PDFs and TMDs

- EMC effect is interpreted as evidence for the medium modification of the bound nucleon wavefunction
  - will be tested in forthcoming experiments – PV DIS, Drell-Yan
  - NuTeV anomaly

- Polarized structure functions of nuclei are potentially interesting
  - polarized EMC effect \([\text{quark spin converted} \rightarrow L_q \text{ in nuclei}]\)
Model Parameters

- **Free Parameters:**
  \[ \Lambda_{IR}, \Lambda_{UV}, M_0, G_\pi, G_s, G_a, G_\omega \text{ and } G_\rho \]

- **Constraints:**
  - \( f_\pi = 93 \text{ MeV}, m_\pi = 140 \text{ MeV} \) \& \( M_N = 940 \text{ MeV} \)
  - \( \int_0^1 dx (\Delta u_v(x) - \Delta d_v(x)) = g_A = 1.267 \)
  - \( (\rho, E_B/A) = (0.16 \text{ fm}^{-3}, -15.7 \text{ MeV}) \)
  - \( a_4 = 32 \text{ MeV} \)
  - \( \Lambda_{IR} = 240 \text{ MeV} \)

- **We obtain [MeV]:**
  - \( \Lambda_{UV} = 644 \)
  - \( M_0 = 400, \ M_s = 690, \ M_a = 990, \ldots \)

- **Can now study a very large array of observables:**
  - e.g. **meson and baryon**: quark distributions, form factors, GPDs, finite temp. and density, neutron stars
**Regularization**

- **Proper-time regularization**

\[
\frac{1}{X^n} = \frac{1}{(n-1)!} \int_0^\infty d\tau \, \tau^{n-1} e^{-\tau X} \quad \longrightarrow \quad \frac{1}{(n-1)!} \int_0^{1/(\Lambda_{IR})^2} d\tau \, \tau^{n-1} e^{-\tau X}
\]

- \(\Lambda_{IR}\) eliminates unphysical thresholds for the nucleon to decay into quarks: \(\rightarrow\) simulates confinement


- E.g.: Quark wave function renormalization

  - \(Z(k^2) = e^{-\Lambda_{UV}(k^2-M^2)} - e^{-\Lambda_{IR}(k^2-M^2)}\)

  \(\rightarrow\) \(Z(k^2 = M^2) = 0 \quad \implies \quad\) no free quarks

- Needed for: nuclear matter saturation, \(\Delta\) baryon, etc

Results: Nuclear Matter

- $\rho_p + \rho_n = \text{fixed} – \text{Differences arise from:}$
  - naive: different number protons and neutrons
  - medium: $p \& n$ Fermi motion and $V_{u(d)}$ differ $\rightarrow u_p(x) \neq d_n(x), \ldots$
For an off-shell nucleon, photon–nucleon vertex given by

\[ \Gamma^\mu_N(p', p) = \sum_{\alpha, \beta = +, -} \Lambda^\alpha(p') \left[ \gamma^\mu f_1^{\alpha\beta} + \frac{i\sigma^{\mu\nu} q_\nu}{2M} f_2^{\alpha\beta} + q^\mu f_3^{\alpha\beta} \right] \Lambda^\alpha(p) \]

In-medium nucleon is off-shell, extremely difficult to quantify effects
- However must understand to fully describe in-medium nucleon

Simpler system: off-shell pion form factors
- relax on-shell constraint \( p'^2 = p^2 = m_\pi^2 \)
- Very difficult to calculate in many approaches, e.g. Lattice QCD

For \( p'^2 = p^2 = m_\pi^2 \) we have \( F_{\pi,1} \to F_\pi \) and \( F_{\pi,2} = 0 \)