RECENT DEVELOPMENTS IN
NJL-JET MODEL: TMD

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OUTLOOK

• Motivation

• Short Overview of the NJL-jet model:
  • Strange quark and Kaons
  • Monte-Carlo approach:
    • Vector mesons, Nucleon-Antinucleon channels, secondary hadrons from the decays of resonances.
  • Transverse Momentum Dependent FF, Hadron TM in SIDIS.
  • Dihadron Fragmentation Functions.

• Future Plans.
EXPLORING HADRON STRUCTURE


- Semi-inclusive deep inelastic scattering (SIDIS): \( eN \rightarrow e h X \)
- Cross-section factorizes into parton distribution and fragmentation functions.

Access to hadron structure:

- Ex., unpolarized cross section is \( \sim \)

\[
\sum_q e_q^2 \int d^2k_\perp f_q^1(x, k_\perp) \pi y^2 \frac{\hat{s}^2 + \hat{u}^2}{Q^4} D_q^h(z, p_\perp)
\]
MOTIVATION

• Providing guidance based on a sophisticated model for applications to problems where phenomenology is difficult/inadequate.

• Unfavored fragmentation functions from the model that goes beyond a single hadron emission approximation.

• Automatically satisfies the sum rules (at the model scale).

• Transverse-momentum dependent (TMD) fragmentations in the same model where structure functions (both unpolarized and polarized) were calculated.
The Quark Jet Model


Assumptions:

• Number Density interpretation
• No re-absorption
• \( \infty \) hadron emissions
The probability of finding mesons $m$ with momentum fraction $z$ in a jet of quark $q$ is given by:

$$D_q^m(z) \, dz = \hat{d}_q^m(z) \, dz + \int_z^1 \hat{d}_q^Q(y) \, dy \cdot D_Q^m(\frac{z}{y}) \, \frac{dz}{y}$$

- **Assumptions:**
  - Number Density interpretation
  - No re-absorption
  - $\infty$ hadron emissions

- **Equation:**
  
  $$D_q^m(z) \, dz = \hat{d}_q^m(z) \, dz + \int_z^1 \hat{d}_q^Q(y) \, dy \cdot D_Q^m(\frac{z}{y}) \, \frac{dz}{y}$$

  - Probability of emitting the meson at link 1
  - Probability of Momentum fraction $y$ is transferred to jet at step 1
  - The probability scales with mom. fraction

**Reference:**
NJL-JET: ELEMENTARY SPLITTING FUNCTIONS FROM NJL

- One-quark truncation of the wavefunction:
  \[ d^m_q(z) : q \rightarrow Qm \quad m = q\bar{Q} \]

- Only 4-point interaction in the Lagrangian
- Lepage-Brodsky (LB)Invariant Mass Cutoff Regularization

\[ u \rightarrow d\pi^+ \]
\[ u \rightarrow sK^+ \]
SOLUTIONS OF THE INTEGRAL EQUATIONS

\[ \pi^+ + K^+ \]

\[ Q_0^2 = 0.2 \text{ GeV}^2 \]

\[ z D_{\pi^+} \]

\[ z D_{K^+} \]

\[ u, d, s \]
THE RATIO OF UNFAVORED TO FAVORED

Model Scale

Evolved

Fit Function - \( f(z) = Nz^\alpha(1 - z)^\beta \)
STRANGENESS EFFECT IN PION


Favored

Unfavored

Q^2 = 4 GeV^2

Empirical
NJL-Jet (Q_0^2 = 0.18 GeV^2)
NJL-Jet
Empirical

Q^2 = 4 GeV^2

Empirical
NJL-Jet (0.2 GeV^2)
NJL-Jet

Q^2 = 4 GeV^2

Empirical
NJL-Jet (0.2 GeV^2)
NJL-Jet

with s quark

no s quark
MONTE-CARLO (MC) APPROACH

- Simulate decay chains to extract number densities.
- Allows for inclusion of TMD and experimental cut-offs.
- Numerically trivially parallelizable (MPI, GPGPU).
Assume Cascade process:

- Sample the emitted hadron according to splitting weight.
- Randomly sample $z$ from input splittings.
- Evolve to sufficiently large number of decay links.
- Repeat for decay chains with the same initial quark.

$$D_{q}^{h}(z) \Delta z = \langle N_{q}^{h}(z, z + \Delta z) \rangle = \frac{\sum_{NSims} N_{q}^{h}(z, z + \Delta z)}{NSims}$$
FRAGMENTATIONS FROM MC STARTING WITH PIONS

• Assume Cascade process:

\[ D_q^h(z) \Delta z = \langle N_q^h(z, z + \Delta z) \rangle = \frac{\sum_{N_{\text{Sims}}} N_q^h(z, z + \Delta z)}{N_{\text{Sims}}} \]

• Sample the emitted hadron according to splitting weight.

• Randomly sample \( z \) from input splittings.

• Evolve to sufficiently large number of decay links.

• Repeat for decay chains with the same initial quark.
DEPENDENCE ON CHAIN CUTOFF

- Restrict the number of emitted hadrons, $N_{Links}$ in MC.

![Graph showing the splitting function and integral equations with different values of $N_{Links}$]

- We reproduce the splitting function and the full solution perfectly.

- The low $z$ region is saturated with just a few emissions.
MORE CHANNELS: VECTOR MESONS

• Calculate quark splittings $d_q^m(z)$ in vector channel:

$$m = \rho^0, \rho^\pm, K^*0, \bar{K^*0}, K^*\pm, \phi$$

• Add the decay of the resonances:

$$dP^{h\rightarrow h_1, h_2}(z_1) = \begin{cases} \frac{C_{h_1, h_2}^h}{8\pi} dz_1 & \text{if } z_1 z_2 m_h^2 - z_2 m_{h_1}^2 - z_1 m_{h_2}^2 \geq 0; \ z_1 + z_2 = 1, \\ 0 & \text{otherwise.} \end{cases}$$
More Channels: Nucleon Anti-Nucleon

- Invoke quark-diquark model for nucleon.

- Calculate splittings $d_q^N(z)$ and $d_{qq}^N(z)$ (quark to nucleon and anti-diquark to anti-nucleon):

- We considered only **scalar** (anti-)diquarks (for now).
Results: Momentum Fractions

\[ \langle z D_u^h (z) \rangle \]
Results: Momentum Fractions

\[ \langle z D^h_u (z) \rangle \]

- Splittings
- NJL-Jet
Results: Momentum Fractions

\[ \langle z \rangle \]

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**Splittings**

**NJL-Jet**

**with Decays**
Results: Momentum Fractions

The Momentum (and Isospin) sumrules satisfied within numerical precision (less than 0.1 %)!
Results: \( Q^2 = 4 \text{ GeV}^2 \)

**Favored**

**Unfavored**
Results: Fragmentations to All Hadrons

$Q^2 = 4 \text{ GeV}^2$

Favored

Unfavored
• TMD splittings: $d(z, p_{\perp}^2)$

• Conserve transverse momenta at each link.

\[
P_{\perp} = p_{\perp} + z k_{\perp}
\]

\[
k_{\perp} = P_{\perp} + k'_{\perp}
\]

• Calculate the Number Density

\[
D^h_{q}(z, P_{\perp}^2) \Delta z \pi \Delta P_{\perp}^2 = \frac{\sum_{N_{Sim}} N^h_{q}(z, z + \Delta z, P_{\perp}^2, P_{\perp}^2 + \Delta P_{\perp}^2)}{N_{Sim}}.
\]
INCLUDING THE TRANSVERSE MOMENTUM

- TMD splittings: \( d(z, p_{\perp}^2) \)
- Conserve transverse momenta at each link.

\[
P_{\perp} = p_{\perp} + z k_{\perp}
\]

\[
k_{\perp} = P_{\perp} + k'_{\perp}
\]

- Calculate the Number Density

\[
D_{q}^{h}(z, P_{\perp}^2) \Delta z \ PI \Delta P_{\perp}^2 = \sum_{NSims} \frac{N_{q}^{h}(z, z + \Delta z, P_{\perp}^2, P_{\perp}^2 + \Delta P_{\perp}^2)}{NSims}.
\]

Approximate \( O(k^2/Q^2) \)
TMD SPLITTING FUNCTIONS

• TMD splittings from the NJL model

• Use dipole cutoff function with LB regularizations

\[ \langle P_{\perp}^2 \rangle \equiv \frac{\int d^2P \ P_{\perp}^2 D(z, P_{\perp}^2)}{\int d^2P \ D(z, P_{\perp}^2)} \]
TMD FRAGMENTATION FUNCTIONS

- Favored
TMD FRAGMENTATION FUNCTIONS

- UNFAVORED
THE TRANSVERSE MOMENTA OF HADRONS IN SIDIS

- Use TMD quark distribution functions calculated in the NJL model (see Ian Cloet’s talk)

- Transfer of the transverse momentum: \[ P_T = P_\perp + z k_\perp \]

- Evaluate \( \langle P_T^2 \rangle \) using MC simulations to calculate the number densities
\[ \langle k_{\perp}^2 \rangle = \frac{\int d^2k_{\perp} k_{\perp}^2 f(x, k_{\perp}^2)}{\int d^2k_{\perp} f(x, k_{\perp}^2)} \]

\[ \langle P_{\perp}^2 \rangle = \frac{\int d^2P_{\perp} P_{\perp}^2 D(z, P_{\perp}^2)}{\int d^2P_{\perp} D(z, P_{\perp}^2)} \]

Using Gaussian Ansatz and:

\[ \langle P_T^2 \rangle = \langle P_{\perp}^2 \rangle + z^2 \langle k_{\perp} \rangle \]

\[ \langle k_{\perp}^2 \rangle = (0.38 \pm 0.06) \text{ GeV}^2 \]

\[ \langle P_{\perp}^2 \rangle = (0.16 \pm 0.01) \text{ GeV}^2 \]
AVERAGE TRANSVERSE MOMENTA

\[ \langle P_T^2 \rangle \equiv \frac{\int d^2 P_T P_T^2 \tilde{D}(z, P_T^2)}{\int d^2 P_T \tilde{D}(z, P_T^2)} \]

\[ P_T = P_\perp + zk_\perp \]

Input: \[ P_T = P_\perp + zk_\perp \]

Output: \[ \langle P_T^2 \rangle = \langle P_\perp^2 \rangle + z^2 \langle k_\perp^2 \rangle \]
AVERAGE TRANSVERSE MOMENTA

\[
\langle P_T^2 \rangle \equiv \frac{\int d^2 P_T \; P_T^2 \tilde{D}(z, P_T^2)}{\int d^2 P_T \; \tilde{D}(z, P_T^2)}
\]

\[<P_T^2> = <P_{\perp}^2> + z^2 <k_{\perp}^2>(0.8)\]

Input: \(P_T = P_{\perp} + z k_{\perp}\)

Output: \(\langle P_T^2 \rangle = \langle P_{\perp}^2 \rangle + z^2 \langle k_{\perp}^2 \rangle\)

D target, Integration over $Q^2$ and $\mathcal{x}$.
DIHADRON FRAGMENTATION FUNCTIONS

\[
D_{q}^{h_1,h_2}(z_1, z_2) = \tilde{d}_q^{h_1}(z_1) \frac{D_{q_1}^{h_2}(\frac{z_2}{1-z_1})}{1-z_1} + \tilde{d}_q^{h_2}(z_2) \frac{D_{q_2}^{h_1}(\frac{z_1}{1-z_2})}{1-z_2} + \sum_Q \int_{z_1+z_2}^1 \frac{d\eta}{\eta^2} \tilde{d}_Q^Q(\eta) D_{Q_1}^{h_1,h_2}(\frac{z_1}{\eta}, \frac{z_2}{\eta})
\]

See Andrew Casey’s Talk on Wednesday at 17:35!
SUMMARY

2009


2010


SUMMARY

- **2009**

- **2010**

- **2011**
  - Coming Soon!

- **2011-2012**
Cheers!
SETTING THE MODEL SCALE
\[ \alpha_s \left( M_Z^2 \right) = 0.118 \]
\[ \alpha_s (M_Z^2) = 0.118 \]

Evolved to 5.2 GeV\(^2\)
\[ \alpha_s \left( M_Z^2 \right) = 0.118 \]

\[ Q_{0\text{NLO}}^2 = 0.2 \text{ GeV}^2 \]

\[ \alpha_s^{\text{NLO}} \left( Q_0^2 \right) = 1.67 \]

Evolved to 5.2 GeV\(^2\)
\[ \alpha_s(M_Z^2) = 0.118 \]

\[ Q_{0\text{NLO}}^2 = 0.2 \text{ GeV}^2 \]

\[ \alpha_s^{NLO}(Q_{0}^2) = 1.67 \]

Evolved to 5.2 GeV$^2$
\[
\alpha_s(M_Z^2) = 0.118
\]

\[
Q_{0NLO}^2 = 0.2\ \text{GeV}^2
\]

\[
\alpha_s^{NLO}(Q_0^2) = 1.67
\]

\[
Q_{0NNLO}^2 = 0.28\ \text{GeV}^2
\]

\[
\alpha_s^{NNLO}(Q_0^2) = 1.59
\]
\[ \alpha_s(M_Z^2) = 0.118 \]

\[ Q_{0\, NLO}^2 = 0.2 \text{ GeV}^2 \]
\[ \alpha_s^{NLO}(Q_0^2) = 1.67 \]

\[ Q_{0\, NNLO}^2 = 0.28 \text{ GeV}^2 \]
\[ \alpha_s^{NNLO}(Q_0^2) = 1.59 \]

\[ \alpha_s(Q_0^2)/2\pi \approx 0.25 \]