SUM RULES FOR LIGHT-BY-LIGHT AND COMPTON SCATTERING

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OUTLINE

• Intro: causality, analyticity and GDH sum rule
• Sum rules for light-by-light scattering: derivation, implications
• Derivative of GDH in Yang-Mills theory
• Analyticity of chiral behavior
INTRODUCTION

causality:

\[ B(t) = \int dt' G(t - t') A(t') \]
\[ G(t - t') = 0, \quad t < t' \]

relativistic version:

\[ B(x) = \int dx' G(x - x') A(x') \]
\[ G(x - x') = 0, \quad (x - x')^2 < 0 \]
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1. \( a(t) = 0 \) if \( t \leq 0 \) and \( a(t) \in L^2 \).
2. \( a(\omega) = F[a(t)] \in L^2 \) if \( \omega \in \mathbb{R} \) and if

\[
    a(\omega) = \lim_{\omega' \to 0} a(\omega + i\omega'),
\]

then \( a(\omega + i\omega') \) is holomorphic if \( \omega' > 0 \).

Statements 1 and 2 are equivalent (Titchmarsh theorem)
The three statements 1, 2, and 3 are mathematically equivalent:

**Theorem 1.** (Titchmarsh)

Titchmarsh's theorem connects, within fairly loose hypotheses, the causality and Kramers-Kronig Relations.

**4.3 Titchmarsh’s Theorem**

Titchmarsh’s theorem [2].

The connection between the mathematical properties of the functions describing the gated function. A rigorous statement of equivalence between causality and relativistic version is a sufficient and necessary condition for strict causality to hold.

However, he made assumptions on the holomorphic behavior of the investigated function. Absorption spectrum. Kronig [5] showed that the existence of dispersion relations, and so of the existence of a strict continuation of the refractive index to the upper half of a complex angular frequency plane, Kramers [4] proved that the principle of relativistic causality: relativistic version:

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\[ G(t - t') = 0, \quad t < t' \]

1. \( a(t) = 0 \) if \( t \leq 0 \) and \( a(t) \in L^2 \).
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then \( a(\omega + i\omega') \) is holomorphic if \( \omega' > 0 \).

\[ \text{Re}\{a(\omega)\} = \frac{1}{\pi} \text{P} \int_{-\infty}^{\infty} \frac{\text{Im}\{a(\omega')\}}{\omega' - \omega} d\omega', \]
\[ \text{Im}\{a(\omega)\} = -\frac{1}{\pi} \text{P} \int_{-\infty}^{\infty} \frac{\text{Re}\{a(\omega')\}}{\omega' - \omega} d\omega'. \]

Statements 1 and 2 are equivalent (Titchmarsh theorem)

e.g., Kramers-Kronig relations

Monday, June 20, 2011
Forward Compton scattering amplitude:

\[ \text{Amp}_{t=0} = f(\omega) \varepsilon' \cdot \varepsilon + g(\omega) i \tilde{S} \cdot (\varepsilon' \times \varepsilon) + \ldots \]

2s + 1 terms \( \leftrightarrow \) \# of e.m. moments

Analyticity: \( \text{Re} g(\omega) = \frac{2\omega}{\pi} \int_0^{\infty} d\omega' \frac{\text{Im} g(\omega')}{\omega'^2 - \omega^2} \)

Unitarity: \( \text{Im} g(\omega) = \frac{\omega}{2} \Delta \sigma(\omega) \)

Low-energy theorem: \( g(\omega) = \frac{e^2 \kappa^2}{4sM^2} \omega + O(\omega^3) \)

branch cuts: \( \pi N, \pi \pi N, \ldots \)

Stability condition: \( \omega_{th} \geq 0 \)
Gerasimov-Drell-Hearn (GDH) sum rule

\[ \frac{e^2}{2M^2} \kappa^2 = \frac{1}{\pi} \int_0^\infty \frac{d\omega}{\omega} \Delta \sigma(\omega) \]

\[ \kappa = (g - 2) s \quad \text{anomalous magnetic moment} \]

\[ \Delta \sigma = \sigma_{1+s} - \sigma_{1-s} \quad \text{doubly-polarized total photoabsorption cross section} \]

(photons circular polarized parallel or anti-parallel to the target’s spin)

Principles/Assumptions:

- **Low-energy theorem for Compton scattering** (gauge-invariance, crossing symmetry, ...)
- **Analyticity** (forward Compton amplitude obeys disp. relations along the production cut)
- **Unitarity** (optical theorem: Im forward Compton amplitude = total photoabsorption)
The verification of the GDH sum rule for the proton:

\[
\frac{e^2}{2M^2} \kappa^2 = \frac{1}{\pi} \int_0^\infty \frac{d\omega}{\omega} \Delta \sigma(\omega)
\]

### Table: Proton Results

<table>
<thead>
<tr>
<th></th>
<th>(E_\gamma) [GeV]</th>
<th>GDH [(\mu b)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>MAID2000 estimate</td>
<td>&lt; 0.2</td>
<td>-28.5 (\pm) 2</td>
</tr>
<tr>
<td>MAMI experiment</td>
<td>0.2 - 0.8</td>
<td>226 (\pm) 5 (\pm) 12</td>
</tr>
<tr>
<td>ELSA experiment</td>
<td>0.8 - 2.9</td>
<td>27.5 (\pm) 2.0 (\pm) 1.2</td>
</tr>
<tr>
<td>Bianchi-Thomas Simula et al. estimate</td>
<td>&gt; 2.9</td>
<td>-14 (\pm) 2</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td><strong>211 (\pm) 15</strong></td>
</tr>
<tr>
<td><strong>GDH sum rule</strong></td>
<td></td>
<td><strong>205</strong></td>
</tr>
</tbody>
</table>
SUM RULES FOR LIGHT-BY-LIGHT
[ V.P. & VANDERHAEGHEN, PRL (2010) ]

\[ M_{\lambda_1 \lambda_2 \lambda_3 \lambda_4} = \varepsilon^{\mu_4 \mu_3 \mu_2 \mu_1}_{\lambda_4 \lambda_3 \lambda_2 \lambda_1}(q_4) \varepsilon^{\mu_3 \mu_2 \mu_1 \mu_0}_{\lambda_3 \lambda_2 \lambda_1 \lambda_0}(q_3) \varepsilon^{\mu_2 \mu_1 \mu_0 \mu_0}_{\lambda_2 \lambda_1 \lambda_1 \lambda_0}(q_2) \varepsilon^{\mu_1 \mu_0 \mu_0 \mu_0}_{\lambda_1 \lambda_1 \lambda_1 \lambda_0}(q_1) \mathcal{M}_{\mu_1 \mu_2 \mu_3 \mu_4} \]

Helicity AMPl. Feynman AMPl.

In the forward direction ( \( t = 0, \ s = 4\omega^2, \ u = -s \) ): 

\[ \mathcal{M}_{\mu_1 \mu_2 \mu_3 \mu_4} = A(s) g_{\mu_4 \mu_2} g_{\mu_3 \mu_1} + B(s) g_{\mu_4 \mu_1} g_{\mu_3 \mu_2} + C(s) g_{\mu_4 \mu_3} g_{\mu_2 \mu_1} , \]

\[ M_{++++}(s) = A(s) + C(s) , \]

\[ M_{++--}(s) = A(s) + B(s) , \]

\[ M_{++--}(s) = B(s) + C(s) . \]

1) Crossing symmetry (1 <-> 3, 2 <-> 4):

\[ M_{+-+-}(s) = M_{++++}(-s) , \quad M_{+++--}(s) = M_{++--}(-s) \]
SUM RULES FOR LIGHT-BY-LIGHT 
(DERIVATION CONTD)

Amplitudes with definite parity under Crossing:
\[ f^{(\pm)}(s) = M_{++++}(s) \pm M_{+-+}(s) \]
\[ g(s) = M_{+++}(s) \]

2) Causality => Analyticity => dispersion relations:
\[
\text{Re} \left\{ \frac{f^{(\pm)}(s)}{g(s)} \right\} = \frac{1}{\pi} \int_{-\infty}^{\infty} \frac{ds'}{s' - s} \text{Im} \left\{ \frac{f^{(\pm)}(s')}{g(s')} \right\},
\]

3) Optical theorem (unitarity):
\[
\text{Im} f^{(\pm)}(s) = -\frac{s}{8} \left[ \sigma_0(s) \pm \sigma_2(s) \right],
\]
\[
\text{Im} g(s) = -\frac{s}{8} \left[ \sigma_{||}(s) - \sigma_{\perp}(s) \right].
\]
\[\sigma_{0,2}(\sigma_{||,\perp}) \text{ Are circularly (linearly) polarized Photon-Photon Fusion cross-sections}\]
Sum rules for light-by-light (derivation contd)

Sum rules:

\[
\text{Re } f^{(+)}(s) = -\frac{1}{2\pi} \int_0^\infty ds' \frac{s'^2 \sigma(s')}{s'^2 - s^2},
\]

\[
\text{Re } f^{(-)}(s) = -\frac{s}{4\pi} \int_0^\infty ds' \frac{s' \Delta \sigma(s')}{s'^2 - s^2},
\]

\[
\text{Re } g(s) = -\frac{1}{4\pi} \int_0^\infty ds' s'^2 \frac{\sigma_{||}(s') - \sigma_{\perp}(s')}{s'^2 - s^2},
\]

\[
\sigma = (\sigma_0 + \sigma_2)/2 = (\sigma_{||} + \sigma_{\perp})/2
\]

\[
\Delta \sigma = \sigma_2 - \sigma_0
\]
SUM RULES FOR LIGHT-BY-LIGHT
(DERIVATION CONT'D)

Sum rules:

\[
\text{Re } f^{(+)}(s) = -\frac{1}{2\pi} \int_{0}^{\infty} ds' \frac{s'^2}{s'^2 - s^2} \sigma(s'), \quad \sigma = \frac{\sigma_0 + \sigma_2}{2} = \frac{\sigma_\| + \sigma_\perp}{2}
\]

\[
\text{Re } f^{(-)}(s) = -\frac{s}{4\pi} \int_{0}^{\infty} ds' \frac{s' \Delta \sigma(s')}{s'^2 - s^2}, \quad \Delta \sigma = \sigma_2 - \sigma_0
\]

\[
\text{Re } g(s) = -\frac{1}{4\pi} \int_{0}^{\infty} ds' \frac{s'^2 \sigma_\|(s') - \sigma_\perp(s')}{s'^2 - s^2},
\]

4) “Low-energy Theorem”: \[\mathcal{L}_{\text{EH}} = c_1(F_{\mu\nu}F^{\mu\nu})^2 + c_2(F_{\mu\nu} \tilde{F}^{\mu\nu})^2,\]
SUM RULES FOR LIGHT-BY-LIGHT
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\[ \text{Re } f^{(+)}(s) = -\frac{1}{2\pi} \int_0^\infty ds' s'^2 \frac{\sigma(s')}{s'^2 - s^2}, \quad \sigma = (\sigma_0 + \sigma_2)/2 = (\sigma_{||} + \sigma_\perp)/2 \]

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\[ \mathcal{L}_{\text{EH}} = c_1 (F_{\mu\nu} F^{\mu\nu})^2 + c_2 (F_{\mu\nu} \tilde{F}^{\mu\nu})^2, \]

\[ f^{(+)}(s) = -2(c_1 + c_2)s^2 + O(s^4) \]

Low-energy expansion

\[ f^{(-)}(s) = O(s^5) \]

\[ g(s) = -2(c_1 - c_2)s^2 + O(s^4) \]
SUM RULES FOR LIGHT-BY-LIGHT

\[ O(s^0) : \quad 0 = \int_0^\infty ds \left[ \sigma_\parallel(s) \pm \sigma_\perp(s) \right] \]

\[ O(s^1) : \quad 0 = \int_0^\infty \frac{ds}{s} \left[ \sigma_2(s) - \sigma_0(s) \right] \]

\[ O(s^2) : \quad c_1 = \frac{1}{8\pi} \int_0^\infty \frac{ds}{s^2} \sigma_\parallel(s) , \]

\[ c_2 = \frac{1}{8\pi} \int_0^\infty \frac{ds}{s^2} \sigma_\perp(s) . \]
SUM RULES FOR LIGHT-BY-LIGHT

\[ O(s^0) : \quad 0 = \int_{0}^{\infty} ds \left[ \sigma_{\parallel}(s) \pm \sigma_{\perp}(s) \right] \]

Gerasimov & Moulin (1976)

\[ O(s^1) : \quad 0 = \int_{0}^{\infty} \frac{ds}{s} \left[ \sigma_2(s) - \sigma_0(s) \right] \]

P. Roy (1974)

Brodsky & SCHMIDT (1995)

\[ O(s^2) : \quad c_1 = \frac{1}{8\pi} \int_{0}^{\infty} \frac{ds}{s^2} \sigma_{\parallel}(s) , \]

\[ c_2 = \frac{1}{8\pi} \int_{0}^{\infty} \frac{ds}{s^2} \sigma_{\perp}(s) \]

Diverges!
\[
0 = \int_{0}^{\infty} ds \frac{\sigma_2(s) - \sigma_0(s)}{s},
\]

\[
\square \equiv X = \begin{array}{c}
\text{\ldots} \\
\end{array}
\]
The low- and high-energy contributions cancel. The result ever, gain a valuable insight into the nature of hadrons.

The expressions for the corresponding helicity

While the role of the sum rules in QED becomes fairly

\[
\sigma_2(s) - \sigma_0(s) \quad ,
\]

\[
0 = \int_0^{\infty} ds \frac{\sigma_2(s) - \sigma_0(s)}{s} ,
\]

\[
\int ds \Delta \sigma/s.
\]


\[
\begin{array}{|c|c|c|c|}
\hline
 & m_M [\text{MeV}] & \Gamma_{\gamma\gamma} [\text{keV}] & \int ds \Delta \sigma/s [\text{nb}] \\
\hline
\pi^0 & 134.98 & (7.8 \pm 0.6) \times 10^{-3} & -195.0 \pm 15.0 \\
\eta & 547.85 & 0.51 \pm 0.03 & -190.7 \pm 11.2 \\
\eta' & 957.66 & 4.30 \pm 0.15 & -301.0 \pm 10.5 \\
\hline
\text{Sum } \eta, \eta' & & & -492 \pm 22 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
 & m_M [\text{MeV}] & \Gamma_{\gamma\gamma} [\text{keV}] & \int ds \Delta \sigma/s \text{ narrow res.} [\text{nb}] & \int ds \Delta \sigma/s \text{ Breit-Wigner} [\text{nb}] \\
\hline
a_2(1320) & 1318.3 & 1.00 \pm 0.06 & 134 \pm 8 & 137 \pm 8 \\
f_2(1270) & 1275.1 & 3.03 \pm 0.35 & 448 \pm 52 & 479 \pm 56 \\
f_2'(1525) & 1525 & 0.081 \pm 0.009 & 7 \pm 1 & 7 \pm 1 \\
\text{Sum } f_2, f_2' & & & 455 \pm 53 & 486 \pm 57 \\
\hline
\end{array}
\]

cancellation of (pseudo)scalar and tensor meson contributions
γ* γ -> M transition form factors

\[
T_{\mu\nu} = i e^2 \varepsilon_{\mu\nu\alpha\beta} q^\alpha q'^\beta F_{\pi^0 \gamma^* \gamma}(Q^2)
\]

\[Q^2 = -q^2 > 0\]

Meson distribution amplitude (DA) \(\varphi(x)\)

Asymptotic DA:

\[\varphi(x) = 6 \times (1 - x)\]

Chiral anomaly

\[f_\pi = 92.4 \text{ MeV}\]
PERTURBATIVE VERIFICATION OF SUM RULES FOR LIGHT-BY-LIGHT

\[ c_1 = \frac{1}{8\pi} \int_0^\infty \frac{ds}{s^2} \sigma_\parallel(s), \]

\[ c_2 = \frac{1}{8\pi} \int_0^\infty \frac{ds}{s^2} \sigma_\perp(s) \]

Böhm & Schuster (1994)

Low-energy Expansion

\[ c_1 = \frac{29\alpha^2}{160M^4} \]

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\[ \sigma_\parallel = \frac{2\pi\alpha^2}{M^2 s^3} \left( s (18M^4 + 3M^2 s + 4s^2) \sqrt{1 - \frac{4M^2}{s}} - 24M^4(s - 3M^2) \text{ArcTanh} \left( \sqrt{1 - \frac{4M^2}{s}} \right) \right) \]
\[ \sigma_\perp = \frac{2\pi\alpha^2}{M^2 s^3} \left( s (6M^4 + 3M^2 s + 4s^2) \sqrt{1 - \frac{4M^2}{s}} - 24M^4(s - M^2) \text{ArcTanh} \left( \sqrt{1 - \frac{4M^2}{s}} \right) \right) \]

V. Pauk (2011)

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Integrate
PERTURBATIVE VERIFICATION OF SUM RULES FOR LIGHT-BY-LIGHT

\[ c_1 = \frac{1}{8\pi} \int_0^\infty \frac{ds}{s^2} \sigma_{\|}(s), \]

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Böhm & Schuster (1994)

Low-energy Expansion

Ghosts, Higgs Sector

Integrate

Unitarity, Causality

V. Pauk (2011)
SUM RULES FOR E.M. MOMENTS OF MASSIVE VECTOR BOSONS

E.M. coupling:
\[
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} - \frac{1}{2} W^*_{\mu\nu} W^{\mu\nu} - M^2 W^*_\mu W^\mu \\
+ i e W^*_\mu A^\mu W^\nu - i e A_\mu W^*_\nu W^{\mu\nu} + e^2 A^2 W^*_\mu W^\mu + i e \ell_1 W^*_\mu W^\nu F^{\mu\nu} \\
+ e \ell_2 \left[ (D^*_\mu W^*_\nu) W^\alpha \partial^\alpha F^{\mu\nu} + W^*_\alpha (D_\mu W^\nu) \partial^\alpha F^{\mu\nu} \right] / (2M^2)
\]

where anomalous magnetic-dipole and electric-Quadrupole moments Are:

\[
\kappa = \frac{1}{2} (\ell_1 - 1), \\
\lambda = \frac{1}{2} (\ell_2 - \ell_1 + 1).
\]

Sum Rules ( \( \omega = (s - M^2)/2 \) ):

\[
O(\omega^1) : \quad \frac{e^2}{2M^2} \kappa^2 = \frac{1}{\pi} \int_0^\infty \frac{ds}{s} \left[ \sigma_2(s) - \sigma_0(s) \right]
\]

\[
O(\omega^2) : \quad 3\kappa(\kappa + \lambda) + \gamma_1 - \gamma_2 = \frac{1}{8\pi} \int_0^\infty \frac{ds}{s^2} \left[ \sigma_\parallel(s) - \sigma_\perp(s) \right],
\]

\( \gamma_{1,2} \) -- transversal spin polarizabilities

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Monday, June 20, 2011
GDH SUM RULE AT ONE LOOP

Consider

$$\frac{e^2}{2M^2} \kappa^2 = \frac{1}{\pi} \int_0^\infty \frac{ds}{s} \left[ \sigma_2(s) - \sigma_0(s) \right]$$

Take a derivative of the sum rule with respect to a.m.m.

Monday, June 20, 2011
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UV-divergent \( \kappa \)

Dispersion Integral

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Take a derivative of the sum rule with respect to a.m.m.

UV-divergent \( \kappa \)

add

Dispersion Integral

\( \kappa = \frac{5\alpha}{3\pi} \)
ANALYTICITY IN PION-MASS SQUARED

[ Ledwig, V.P. & Vanderhaeghen, PLB (2010) ]

Motivation: chiral perturbation theory and lattice QCD, which compute the pion-mass dependence of hadron properties

\[ t = m_{\pi}^2 \]

\[ f(m_{\pi}^2) = -\frac{1}{\pi} \int_{-\infty}^{0} dt \frac{\text{Im} f(t)}{t - m_{\pi}^2 + i0^+} \]

Verified for nucleon mass, a.m.m., polarizabilities at order p^3.
DELTA(1232) RESONANCE

\[ f(m_\pi^2) = -\frac{1}{\pi} \int_{-\infty}^{\Delta^2} dt' \frac{\text{Im} f(t')}{t' - m_\pi^2 + i0^+}. \]

with \( \Delta = M_\Delta - M_N \), the Delta-nucleon mass difference.
DELTA(1232) RESONANCE

\[ f(m^2_\pi) = -\frac{1}{\pi} \int_{-\infty}^{\Delta^2} dt' \frac{\text{Im} f(t')}{t' - m^2_\pi + i0^+}. \]

with \( \Delta = M_\Delta - M_N \), the Delta-nucleon mass difference.

Magnetic moment:

- \( \Delta^{++} \)
- \( \Delta^+ \)
- \( \Delta^0 \)

Curves - chiral EFT calculation:

- Real parts
- Imag. parts

quenched lattice points:

- Leinweber, PRD (1992)
- Cloet, Leinweber, Thomas, PLB (2003)

SUMMARY

• Sum rules express a quantum phenomenon in terms of an integral over a classical quantity – (polarized) cross sections based on general principles (unitarity, causality, crossing) – non-perturbative
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• Motivation to measure polarized gamma-gamma cross sections (possibly at BES)

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• Analyticity of the chiral behavior observed in chiral perturbation theory